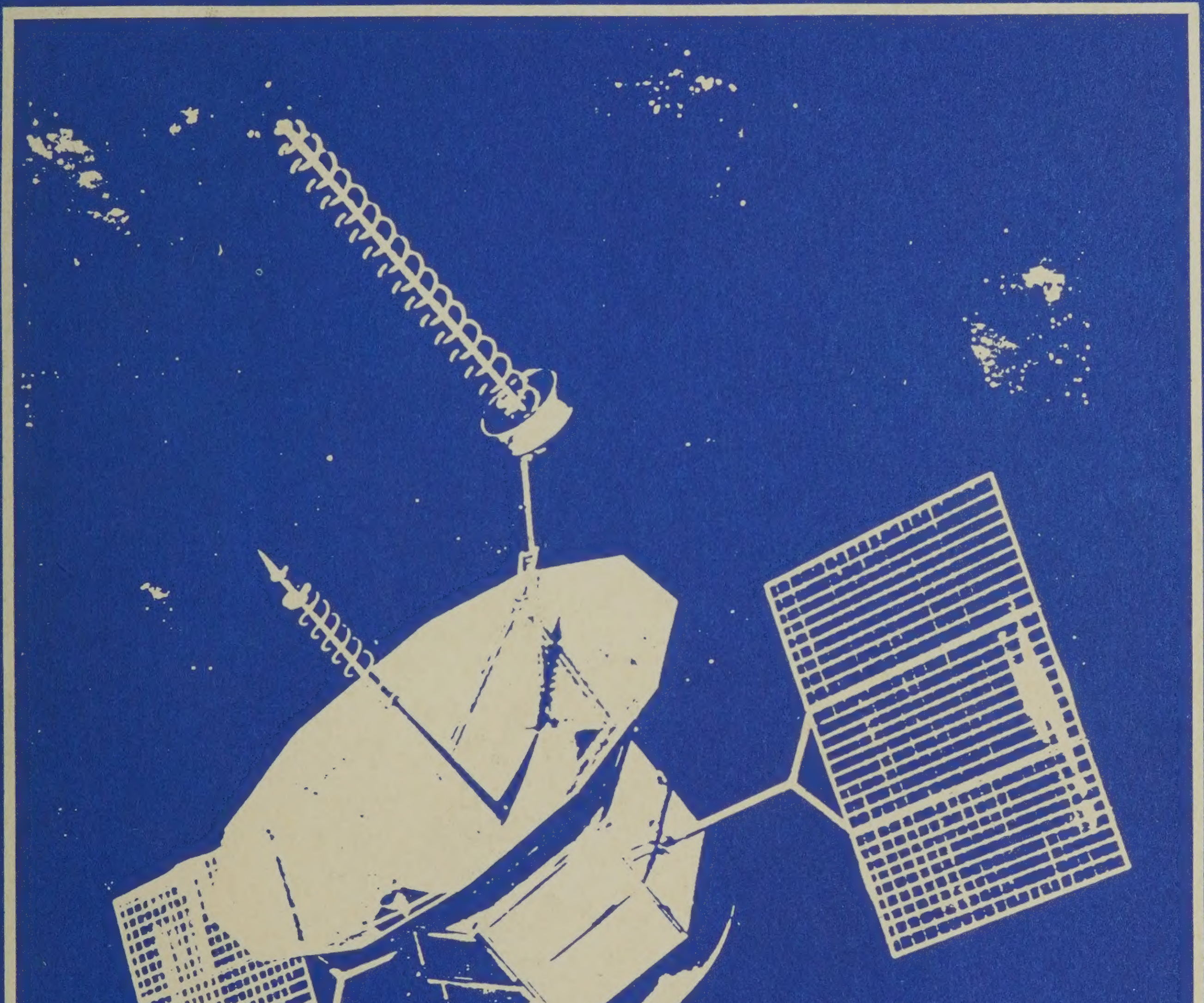
Instructor's Manual to Accompany

ANTENNAS

SECOND EDITION



JOHN D. KRAUS

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PREFACE

This Instructors' Manual provides solutions to most of the problems in the Second Edition of ANTENNAS. All problems are solved for which answers appear in Appendix D of the text and in addition solutions are given for a large fraction of the other problems. Including multiple parts, there are nearly 600 problems in the text and solutions are presented here for about 85 percent of them.

Many of the problem titles are supplemented by key words or phrases alluding to the solution procedure. Answers are boxed. Many tips on solutions are included which can be passed on to students.

Although an objective of problem solving is to obtain an answer, I have endeavored to also provide insights as to how many of the problems relate to engineering situations in the real world.

The Manual includes an index to assist in finding problems by topic or principle and to facilitate finding closely-related problems.

The Manual was prepared with the assistance of Dr. Erich Pacht of the Ohio State University Radio Observatory.

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TEXT AIDS

Note that the *running head* at the upper right of each right-hand text page lists chapter and section number (with section title) which facilitates locating equations.

Note that *video tape* teaching aids are listed on page 865 of the text.

Computer programs are given (in the text) or can be written to facilitate the solution of many of the problems in particular

4-4, 4-22, 4-24, 4-26, 4-31, 4-32, 4-35, 4-36, 4-38, 4-40, 4-47, 7-2, 7-3, 9-2, 9-6, 10-1, 10-2, 11-1, 11-3, 11-8, 11-18, 11-34, 12-19, 16-13, 16-15, 16-16, 16-19, 18-13.

The time required to write a new program may be worthwhile not only on long calculations but also on shorter ones which are repeated with changes only in some parameters.

TABLE OF CONTENTS

PROBLEM SOLUTIONS

CHAPTER 2. BASIC ANTENNA CONCEPTS

2-1 **Directivity.** Radiation intensity.

From (2-8-1)

$$D = \frac{U(\theta,\phi)_{max}}{U_{av}}$$

where

$$U(\theta,\phi)_{max} = S(\theta,\phi)_{max}\ r^2$$

$$U_{av} = \frac{1}{4\pi}\int\int_{4\pi} U(\theta,\phi)\ d\Omega$$

$$U(\theta,\phi) = S(\theta,\phi)\ r^2$$

$$S(\theta,\phi) = \frac{E(\theta,\phi)E^*(\theta,\phi)}{Z}$$

Therefore

$$D = \frac{\dfrac{E(\theta,\phi)_{max}E^*(\theta,\phi)_{max}}{Z}\ r^2}{\dfrac{1}{4\pi}\displaystyle\int\int_{4\pi}\frac{E(\theta,\phi)E^*(\theta,\phi)}{Z}\ r^2\ d\Omega} \qquad \text{q.e.d.}$$

Note that $r^2 =$ area/steradian

so $U = Sr^2$

or

$$\frac{\text{watts}}{\text{steradian}} = \frac{\text{watts}}{\text{meter}^2} \times \text{meter}^2$$

2-2 Directivity. Aperture efficiency.

If the field over the aperture is uniform, the directivity is a maximum (= D_m) and the power radiated is P'. For an actual aperture distribution, the directivity is D and the power radiated is P. Equating effective powers

$$D_m P' = D P$$

$$D = D_m \frac{P'}{P} = \frac{4\pi}{\lambda^2} A_p \frac{\frac{E_{av}E^*_{av}}{Z} A_p}{\iint_{A_p} \frac{E(x,y)E^*(x,y)}{Z} dxdy}$$

where $$E_{av} = \frac{1}{A_p} \iint_{A_p} E(x,y)\ dxdy$$

therefore $$D = \frac{4\pi}{\lambda^2} \frac{\iint_{A_p} E(x,y)\ dxdy \iint_{A_p} E^*(x,y)\ dxdy}{\iint_{A_p} E(x,y)E^*(x,y)\ dxdy} \qquad \text{q.e.d.}$$

where

$$\frac{E_{av}E^*_{av}A_p}{\iint_{A_p} E(x,y)E^*(x,y)\ dxdy} = \frac{E_{av}E^*_{av}}{\frac{1}{A_p}\iint E(x,y)E^*(x,y)\ dxdy}$$

$$= \frac{E^2_{av}}{(E^2)_{av}} = \epsilon_{ap} = \frac{A_e}{A_p}$$

2-3 Effective aperture. Beam area.

$$\Omega_A \cong \theta_{HP}\phi_{HP} = 30^\circ \times 35^\circ$$

$$A_{em} = \frac{\lambda^2}{\Omega_A} \cong \frac{57.3^2}{30^\circ \times 35^\circ} \lambda^2 = \boxed{3.1\ \lambda^2}$$

***2-4** **Effective aperture.** Directivity.

$$D = 4\pi A_{em}/\lambda^2$$

$$A_{em} = \frac{D\lambda^2}{4\pi} = \frac{900}{4\pi}\lambda^2 = \boxed{71.6\ \lambda^2}$$

2-5 **Received power.** Friis formula.

$$\lambda = c/f = 3 \times 10^8/10^9 = 0.3 \text{ m}$$

$$A_{et} = \frac{D_t\ \lambda^2}{4\pi}, \qquad A_{er} = \frac{D_r\ \lambda^2}{4\pi}$$

$$P_r = P_t \frac{A_{et}\ A_{er}}{r^2\lambda^2} = P_t \frac{D_t\ \lambda^2\ D_r\ \lambda^2}{(4\pi)^2\ r^2\ \lambda^2}$$

$$= 150\ \frac{316 \times 0.3^2 \times 100}{(4\pi)^2\ 500^2} = 0.0108 \text{ W} = \boxed{10.8 \text{ mW}}$$

***2-6** **Spacecraft link over 100 Mm.** Friis formula.

$$\lambda = c/f = 3 \times 10^8/2.5 \times 10^9 = 0.12 \text{ m}$$

$$A_{et} = A_{er} = \frac{D\ \lambda^2}{4\pi}$$

$$P_r(\text{required}) = 100 \times 10^{-12} = 10^{-10} \text{ W}$$

$$P_t = P_r \frac{r^2\ \lambda^2}{A_{et}^2} = P_r \frac{(4\pi)^2\ r^2\ \lambda^2}{D^2\ \lambda^4}$$

$$= P_r \frac{r^2\ (4\pi)^2}{D^2\ \lambda^2} = 10^{-10} \frac{10^{16}\ (4\pi)^2}{10^6\ 0.12^2}$$

$$= 10966 \text{ W} \cong \boxed{11 \text{ kW}}$$

<u>2-7</u> Spacecraft link over 3 Mm. Friis formula.

$$\lambda = c/f = 3 \times 10^8/2 \times 10^9 = 0.15 \text{ m}$$

$$A_{et} = A_{er} = \frac{D\,\lambda^2}{4\pi}$$

$$P_r = 100 \times 10^{-12} = 10^{-10} \text{ W}$$

$$P_t = P_r \frac{r^2\,\lambda^2}{A_{et}\;A_{er}} = P_r \frac{(4\pi)^2\,r^2\,\lambda^2}{D\,\lambda^2\,\lambda^2}$$

$$= 10^{-10} \frac{(4\pi)^2\;9 \times 10^{12}}{4 \times 10^4 \times 0.15^2} = \boxed{158 \text{ W}}$$

<u>2-8</u> Mars link. Friis formula.

(a) $$\lambda = c/f = 3 \times 10^8/2.5 \times 10^9 = 0.12 \text{ m}$$

$$P_r(\text{earth}) = 10^{-19} \times 5 \times 10^6 = 5 \times 10^{-13} \text{ W}$$

$$P_r(\text{Mars}) = 10^{-17} \times 5 \times 10^6 = 5 \times 10^{-11} \text{ W}$$

Take $A_e(\text{Mars}) = (1/2)\,\pi\,1.5^2 = 3.5 \text{ m}^2$ $\quad(\epsilon_{ap} = 0.5)$

Take $P_t(\text{Mars}) = 1 \text{ kW}$

Take $A_e(\text{earth}) = (1/2)\,\pi\,15^2 = 350 \text{ m}^2$ $\quad(\epsilon_{ap} = 0.5)$

$$P_t(\text{earth}) = P_r(\text{Mars}) \frac{r^2\,\lambda^2}{A_{et}(\text{earth})\;A_{er}(\text{Mars})}$$

$$P_t(\text{earth}) = 5 \times 10^{-11} \frac{(360 \times 3 \times 10^8)^2\;0.12^2}{3.5 \times 350}$$

$$= 6.9 \text{ MW}$$

To reduce the required earth station power, take the earth station antenna

$$A_e = (1/2)\,\pi\,50^2 = \boxed{3927 \text{ m}^2}$$

2-8 continued

so

$$P_t(\text{earth}) = 6.9 \times 10^6 \, (15/50)^2 = \boxed{620 \text{ kW}}$$

$$P_r(\text{earth}) = P_t(\text{Mars}) \frac{A_{et}(\text{Mars}) \, A_{er}(\text{earth})}{r^2 \, \lambda^2}$$

$$= 10^3 \frac{3.5 \times 3930}{(360 \times 3 \times 10^8)^2 \, 0.12^2}$$

$$= 8 \times 10^{-14} \text{ W}$$

which is about 16% of the required 5×10^{-13} W. The required 5×10^{-13} W could be obtained by increasing the Mars transmitter power by a factor of 6.3. Other alternatives would be (1) to reduce the bandwidth (and data rate) reducing the required value of P_r or (2) to employ a more sensitive receiver.

As discussed in Sec. 17-3, the <u>noise power</u> of a receiving system is a function of its system temperature T and bandwidth B as given by

$$P = kTB$$

where k = Boltzmann's constant = 1.38×10^{-23} JK^{-1}

For B = 5×10^6 Hz (as given in this problem)

and T = 50 K (an attainable value),

$$P(\text{noise}) = 1.38 \times 10^{-23} \times 50 \times 5 \times 10^6 = 3.5 \times 10^{-15} \text{ W}$$

The received power (8×10^{-14} W) is about 20 times this noise power, which is probably sufficient for satisfactory communication. Accordingly, with a 50 K receiving system temperature at the earth station, a Mars transmitter power of 1 kW is adequate.

continued

2-8 continued

(b) The given Jupiter distance is 40/6 = 6.7 times that to Mars, which makes the required transmitter powers 6.7^2 = 45 times as much or the required receiver powers 1/45 as much.

Neither appears feasable. But a practical solution would be to reduce the bandwidth for the Jupiter link by a factor of about 50, making $B = (5/50) \times 10^6 = 100$ kHz.

---

*2-9 Moon link. Friis formula.

$$\lambda = c/f = 3 \times 10^8 / 1.5 \times 10^9 = 0.2 \text{ m}$$

From (7-4-7) the directivity of the moon helix is given by

$$D = 12 \times 5 = 60$$

and

$$A_{et}(\text{moon}) = \frac{D\lambda^2}{4\pi}$$

From Friis formula

$$A_{er} = \frac{P_r\, r^2\, \lambda^2}{P_t\, A_{et}} = \frac{P_r (4\pi)\, r^2\, \lambda^2}{P_t\, D\, \lambda^2}$$

$$= \frac{10^{-14}\,(3 \times 10^8 \times 1.27)^2\, 4\pi}{2 \times 60}$$

$$= \boxed{152 \text{ m}^2 \quad \text{RCP}}$$ or about 14 m diameter

---

2-10 Crossed dipoles. Polarization.

(a) $\boxed{\text{LP}}$ (b) $\boxed{\text{CP}}$

(c) From (2-36-3)

$$\sin 2\epsilon = \sin 2\gamma \ \sin \delta$$

where $\gamma = \tan^{-1}(E_2/E_1) = 45^\circ$

$\delta = 45^\circ$

$\epsilon = 22\tfrac{1}{2}^\circ$

$AR = \cot \epsilon = 1/\tan \epsilon = \boxed{2.41 \quad (\text{EP})}$

*2-11 **Two LP waves.** Polarization.

(a) From (2-34-8) $\quad AR = 3/2 = \boxed{1.5}$

(b) $\boxed{90^\circ}$

(c) At $t = 0$, $E = E_x$

At $t = T/4$, $E = -E_y$

Therefore rotation is $\boxed{\text{CW}}$

2-12 **Two EP waves.** Superposition.

$$E_y = E_y' + E_y'' = 2 \cos \omega t + \cos \omega t = 3 \cos \omega t$$

$$E_x = E_x' + E_x'' = 6 \cos (\omega t + \pi/2) + 3 \cos (\omega t - \pi/2)$$

$$= -6 \sin \omega t + 3 \sin \omega t = -3 \sin \omega t$$

(a) E_x and E_y are in phase quadrature and

$AR = 3/3 = \boxed{1\ (\text{CP})}$

(b) At $t = 0$, $\mathbf{E} = \hat{\mathbf{y}}3$

At $t = T/4$, $\mathbf{E} = -\hat{\mathbf{x}}3$

Therefore rotation is $\boxed{\text{CCW}}$

---

*2-13 **Two LP components.** Tilt angle.

$\gamma = \tan^{-1} (E_2/E_1) = 45^\circ$

$\delta = 72^\circ$

From (2-36-3), $\epsilon = 36^\circ$

Therefore $AR = 1/\tan \epsilon = \boxed{1.38}$ $\quad$ (b)

continued

*2-13 continued

From (2-36-3), $\sin 2\tau = \tan 2\epsilon / \tan \delta$

or $\tau = \boxed{45^\circ}$ (c)

---

*2-14 Two LP components. Poincaré sphere.

$\gamma = \tan^{-1} 2 = 63.4^\circ$

$\delta = 72^\circ$

$\epsilon = 24.8^\circ$ and AR = $\boxed{2.17}$ (b)

$\tau = \boxed{11.2^\circ}$ (c)

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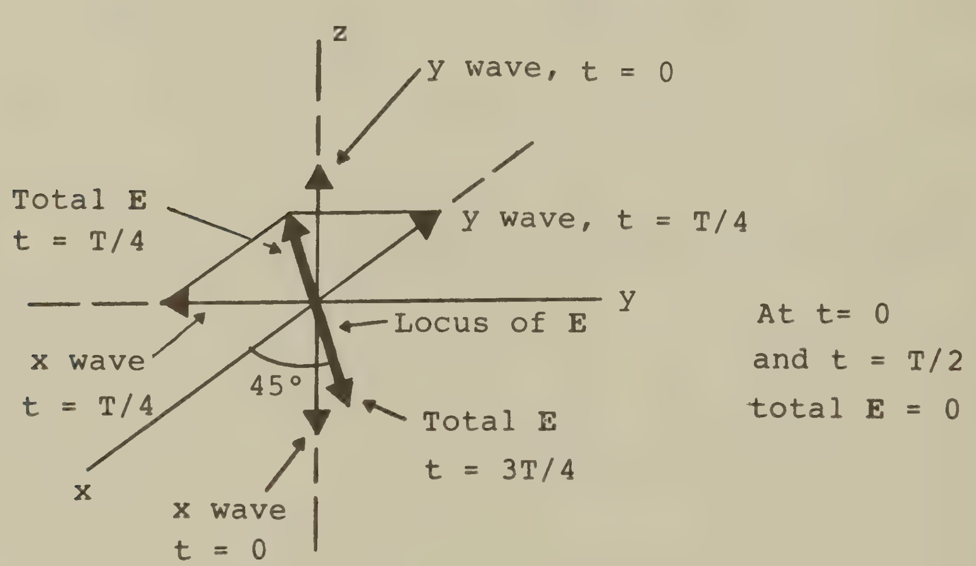
*2-15 Two CP waves.

Resolve 2 waves into components or make sketch as shown. It is assumed that the waves have equal magnitude.



Locus of E is a straight line in xy plane at an angle of 45° with respect to x (or y) axis. ans.

2-17 Spaceship near moon.

(a) $PV \text{ at earth} = \dfrac{P_t}{(4\pi)\ r^2} = \dfrac{10}{4\pi(380 \times 10^6)^2}$

$= 5.5 \times 10^{-18}\ W = \boxed{5.5\ aW}$

(b) $PV = S = E^2/Z$ or $E = (SZ)^{\frac{1}{2}}$

or $E = (5.5 \times 10^{-18} \times 377)^{\frac{1}{2}} = 45 \times 10^{-9} = \boxed{45\ nV}$

(c) $t = r/c = 380 \times 10^6/3 \times 10^8 = \boxed{1.27\ s}$

(d) Photon = hf where $h = 6.63 \times 10^{-34}$ J s

$= 6.63 \times 10^{-34} \times 2 \times 10^9 = 1.3 \times 10^{-24}$ J

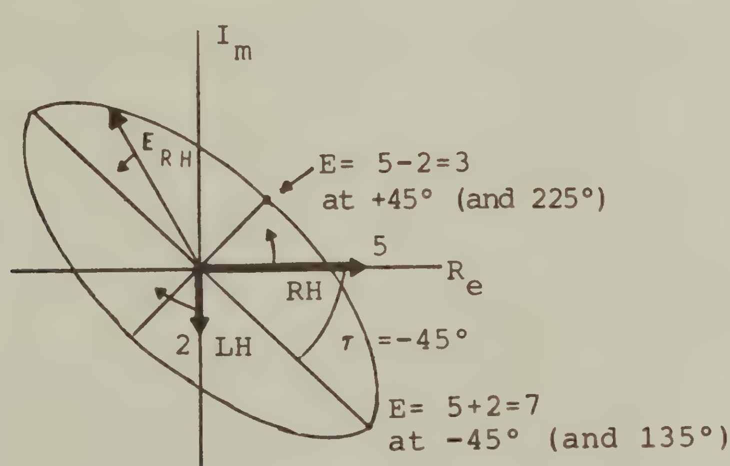
This is the energy of a 2.5 MHz photon.

From (a), $PV = 5.5 \times 10^{-18}\ J\ s^{-1}\ m^{-2}$

Therefore,

$\text{number of photons} = \dfrac{5.5 \times 10^{-18}}{1.3 \times 10^{-24}} = \boxed{4.2 \times 10^6\ m^{-2}\ s^{-1}}$

***2-18** CP waves. Superposition.



(a) $AR = \dfrac{2+5}{2-5} = -7/3 = \boxed{-2.33}$ [Note: minus sign for RH (right-handed) polarization]

continued

*2-18 continued

(b) τ = $\boxed{-45^\circ}$ from diagram

(c) $\boxed{\text{RH}}$ since **E** rotates counterclockwise as a function of time.

2-19 EP wave.

(a) $AR = 3/2 = \boxed{1.5}$

(b) $\tau = \boxed{0^\circ}$

(c) $\boxed{\text{CW}}$ $\boxed{\text{LEP}}$

---

*2-20 CP waves.

(a) $AR = \dfrac{2 + 3}{2 - 3} = \boxed{-5}$

(b) $\boxed{\text{REP}}$

(c) $PV = \dfrac{E_L^2 + E_R^2}{Z} = \dfrac{4 + 9}{377} = 0.034 \text{ W m}^{-2}$

$= \boxed{34 \text{ mW m}^{-2}}$

2-21 EP waves. Superposition.

(a) $E_x = 5 \cos \omega t + 3 \cos \omega t + 4 \cos \omega t = 12 \cos \omega t$

$E_y = 3 \sin \omega t + 3 \sin \omega t - 4 \sin \omega t = 2 \sin \omega t$

$AR = 12/2 = \boxed{6}$

(b) Since E_x and E_y are in time-phase quadrature with

$E_x(\max) > E_y(\max)$, $\tau = 0^\circ$.

Or from (2-36-3)

$\sin 2\tau = \tan 2\epsilon / \tan \delta$

$\epsilon = \tan^{-1}(1/AR) = 9.46^\circ$

but $\delta = 90^\circ$ so

$\tan \delta = \infty$

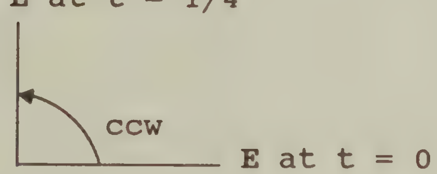
Therefore $\tau = \boxed{0^\circ}$

(c) At $t = 0$, $E_x = 12$, $E_y = 0$

at $t = T/4$ $(\omega t = 90^\circ)$, $E_x = 0$, $E_y = 2$

Therefore rotation is CCW, so polarization is right

elliptical, $\boxed{\text{REP}}$



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*2-22 CP waves. Superposition.

(a) $AR = \dfrac{E_l + E_r}{E_l - E_r} = \dfrac{4+2}{4-2} = \dfrac{6}{2} = \boxed{3}$

(b) When $\omega t = 0$, $E_r = 2 \underline{/0^\circ}$ and $E_l = 4 \underline{/-45^\circ}$

When $\omega t = -22\frac{1}{2}^\circ$, $E_r = 2 \underline{/-22\frac{1}{2}^\circ}$ and $E_l = 4 \underline{/-22\frac{1}{2}^\circ}$

so that $E_l + E_r = E_{max} = 6 \underline{/-22\frac{1}{2}^\circ}$ or $\tau = \boxed{-22\frac{1}{2}^\circ}$

continued

*2-22 continued

Note that the rotation directions are opposite for E_r and E_l so that for $-\omega t$, $E_r = 2 \underline{/-\omega t}$ but $E_l = \underline{/+\omega t}$.

Also, τ can be determined analytically by combining the waves into an E_x and E_y component with values of

$$E_x = 5.60 \underline{/-30.4^\circ}$$

$$E_y = 2.95 \underline{/16.3^\circ}$$

from which $\delta = -46.7^\circ$.

Since from (a) AR = 3, ϵ can be determined and from (2-36-3) the tilt angle $\tau = -22.5^\circ$.

(c) $E_l > E_r$ so rotation is $\boxed{\text{CW (LEP)}}$

2-23 **More power with CP. Poynting vector.**

From (2-35-13) we have for rms fields that

$$PV = S_{av} = \frac{E_1^2 + E_2^2}{Z_o}$$

For LP, E_2 (or E_1) = 0, so $S_{av} = \dfrac{E_1^2}{Z_o}$

For CP, $E_1 = E_2$, so $S_{av} = \dfrac{2E_1^2}{Z_o}$

Therefore $\boxed{S_{CP} = 2\, S_{LP}}$

2-24 **PV constant for CP.**

$E_{cp} = E_x \cos \omega t + E_y \sin \omega t$

where $E_x = E_y = E_o$

2-24 **continued**

$$\left| E_{cp} \right| = (E_0^2 \cos^2 \omega t + E_0^2 \sin^2 \omega t)^{1/2}$$

$$= E_o(\cos^2 \omega t + \sin^2 \omega t)^{1/2} = E_o \quad \text{(a constant)}$$

Therefore PV or S(instantaneous) = $\boxed{\dfrac{E_0^2}{Z} \quad \text{(a constant)}}$

***2-25** **EP wave power.** Poynting vector.

From (2-35-12)

$$S_{av} = (1/2)\ Z\ (H_1^2 + H_2^2) = (1/2)\ 377\ (\mu_r/\epsilon_r)^{1/2}\ (H_1^2 + H_2^2)$$

$$= (1/2)\ 377\ (2/5)^{1/2}\ (3^2 + 4^2) = 2980 \text{ W m}^{-2}$$

$P = A\ S_{av} = 5 \times 2980 = 14902 \text{ W} = \boxed{14.9 \text{ kW}}$

2-26 **Circular-depolarization ratio.** CP components.

Any wave may be resolved into 2 circularly-polarized components of opposite hand, E_r and E_l for an axial ratio

$$AR = \frac{E_{max}}{E_{min}} = \left| \frac{E_r + E_l}{E_r - E_l} \right|$$

from which the circular depolarization ratio

$$R = \frac{E_l}{E_r} = \frac{AR - 1}{AR + 1}$$

Thus, for pure circular polarization, AR = 1 and there is zero depolarization (R = 0), while for pure linear polarization AR = ∞ and the depolarization ratio is unity (R = 1).

When AR = 3, R = 1/2.

2-27 **Superluminal phase velocity near dipole.**

(b) Two examples are in waveguides and on small helices (see Fig. 7-28 in text).

CHAPTER 3

POINT SOURCES

*3-1 Directivity.

(a)

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{\pi}\int_0^{2\pi} \frac{U}{U_m}\, d\Omega}$$

If $U = U_m \sin\theta \sin^2\phi$, for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$

$$D = \frac{4\pi}{\int_0^{\pi}\int_0^{\pi} \sin\theta \sin^2\phi \underbrace{\sin\theta\, d\theta d\phi}_{d\Omega}}$$

$$D = \frac{4\pi}{\int_0^{\pi}\int_0^{\pi} \sin^2\theta \sin^2\phi\, d\theta d\phi}$$

where $\int_0^{\pi} \sin^2\theta\, d\theta = \left.\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{\pi} = \frac{\pi}{2}$

and $\int_0^{\pi} \sin^2\phi\, d\phi = \frac{\pi}{2}$

Therefore,

$$D = \frac{4\pi}{\pi^2/4} = 5.09 \cong \boxed{5.1}$$

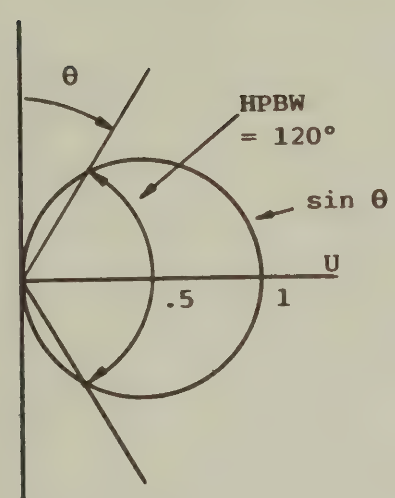
If $U = U_m \sin\theta \sin^3\phi$, for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$

$$D = \frac{4\pi}{\int_0^\pi \int_0^\pi \sin^2\theta \sin^3\phi \, d\theta d\phi} = \frac{4\pi}{\frac{\pi}{2}\ \frac{4}{3}} = \boxed{6}$$

If $U = U_m \sin^2\theta \sin^3\phi$, for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$

$$D = \frac{4\pi}{\int_0^\pi \int_0^\pi \sin^3\theta \sin^3\phi \, d\theta d\phi} = \frac{4\pi}{\left(\frac{4}{3}\right)^2} = \boxed{7.07}$$

(b) Half-power level = half radiation intensity level



For $\sin\theta$, U-pattern (as in sketch)

$$HPBW = 2[90^\circ - \sin^{-1}(1/2)] = 120^\circ$$

For $\sin^2\theta$ (or $\sin^2\phi$),

$$HPBW = 2[90^\circ - \sin^{-1}(1/\sqrt{2})] = 90^\circ$$

For $\sin^3\phi$,

$$HPBW = 2[90^\circ - \sin^{-1}(1/\sqrt[3]{2})]$$
$$= 75^\circ$$

Therefore, for $\sin\theta \sin^2\phi$ pattern,

$$D = \frac{41\,000^{\square}}{\theta_{HP}\,\phi_{HP}} = \frac{41\,000^{\square}}{120^\circ \times 90^\circ} = \boxed{3.80}$$ (vs. 5.1 exact)

(1.3 dB difference)

continued

*3-1 continued

For $\sin\theta \sin^3\phi$ pattern,

$$D = \frac{41\ 000}{120 \times 75} = \boxed{4.56} \quad \text{(vs. 6 exact)}$$

(1.2 dB difference)

For $\sin^2\theta \sin^3\phi$ pattern,

$$D = \frac{41\ 000}{90 \times 75} = \boxed{6.07} \quad \text{(vs. 7.07 exact)}$$

(0.7 dB difference)

3-2 Directivity.

If $U = U_m \cos^n\theta$

$$D = \frac{4\pi}{2\pi \int_0^{\pi/2} \sin\theta \cos^n\theta \, d\theta} = \frac{2}{-\left. \dfrac{\cos^{n+1}\theta}{n+1} \right]_0^{\pi/2}}$$

$$= \boxed{2(n+1)} \quad \text{q.e.d.}$$

---

*3-3 Solar power.

(a)

$$S = \frac{2.2 \text{ g cal min}^{-1} \text{ cm}^{-2}}{14.3 \text{ g cal min}^{-1}} = 0.1539 \text{ W cm}^{-2} = \boxed{1539 \text{ W m}^{-2}}$$

(b)

$$P(\text{sun}) = S \times 4\pi r^2 = 1539 \times 4\pi \times 1.49^2 \times 10^{22} \text{ W}$$

$$= \boxed{4.29 \times 10^{26} \text{ W}}$$

(c)

$$S = E^2/Z_o, \quad E = (SZ_o)^{\frac{1}{2}} = (1539 \times 377)^{\frac{1}{2}} = \boxed{762 \text{ V m}^{-1}}$$

3-4 **Directivity and minor lobes.**

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\Omega_M + \Omega_m}$$

where Ω_A = total beam area

Ω_M = main lobe beam area

Ω_m = minor lobe beam area

as $\Omega_M \longrightarrow 0$, $\Omega_A \longrightarrow \Omega_m$, so $D = \boxed{4\pi/\Omega_m \text{ (a constant)}}$

3-5 **Directivity by integration.**

Exact values for (a), (b) and (c) are 4, 6 and 8.

3-6 **Directivity.**

Assuming a unidirectional pattern

$(0 \le \theta \le \pi/2)$, $D \cong \boxed{24}$

CHAPTER 4

ARRAYS OF POINT SOURCES

4-2 Four sources in square array.

(a) $E_n(\phi) = \cos(\beta d \cos\phi) - \cos(\beta d \sin\phi)$

4-7 Eight source D-T distribution.

(a) 0.14, 0.42, 0.75, 1.00, 1.00, 0.75, 0.42, 0.14

(b) Max. at: $\pm 21^\circ$, $\pm 27^\circ$, $\pm 36^\circ$, $\pm 48^\circ$, $\pm 61^\circ$, $\pm 84^\circ$,
$\pm 96^\circ$, $\pm 119^\circ$, $\pm 132^\circ$, $\pm 144^\circ$, $\pm 153^\circ$, $\pm 159^\circ$

Nulls at: $\pm 18^\circ$, $\pm 23^\circ$, $\pm 32^\circ$, $\pm 42^\circ$, $\pm 54^\circ$, $\pm 71^\circ$,
$\pm 109^\circ$, $\pm 126^\circ$, $\pm 138^\circ$, $\pm 148^\circ$, $\pm 157^\circ$, $\pm 162^\circ$

(d) HPBW = $\boxed{12^\circ}$

4-11 Two-source end-fire array.

(a) D = $\boxed{2}$

4-14 Field and phase patterns.

See Figures 4-16 and 4-17.

4-24 Twelve source end-fire array.

(b) D = $\boxed{17}$ (c) D = $\boxed{10}$

4-26 **Twelve source end-fire array with increased directivity.**

(b) $D = \boxed{26}$ (c) $D = \boxed{35}$

4-31 **Directivity of ordinary end-fire array.** Change of variable.

It is assumed that the array has a uniform spacing d between the isotropic sources. The beam area

$$\Omega_A = \frac{1}{n^2}\int_0^{2\pi}\int_0^{\pi}\left[\frac{\sin(n\Psi/2)}{\sin(\Psi/2)}\right]^2 \sin\theta\, d\theta d\phi \tag{1}$$

where θ = angle from array axis.

The pattern is not a function of ϕ so (1) reduces to

$$\Omega_A = \frac{2\pi}{n^2}\int_0^{\pi}\left[\frac{\sin(n\Psi/2)}{\sin(\Psi/2)}\right]^2 \sin\theta\, d\theta \tag{2}$$

where $\Psi/2 = \pi d_\lambda(\cos\theta - 1)$ (2.1)

Differentiating

$$d\frac{\Psi}{2} = \pi d_\lambda \sin\theta\, d\theta \tag{3}$$

or

$$\sin\theta\, d\theta = \frac{1}{\pi d_\lambda}\frac{\Psi}{2} \tag{4}$$

and introducing (4) in (2)

$$\Omega_A = \frac{2}{n^2 d_\lambda}\int_0^{2\pi d_\lambda}\left[\frac{\sin(\Psi/2)}{\sin(\Psi/2)}\right]^2 d\frac{\Psi}{2} \tag{5}$$

Note new limits with change of variable from θ to $\Psi/2$.

When $\theta = 0$, $\Psi/2 = 0$ and when $\theta = \pi$, $\Psi/2 = 2\pi d_\lambda$.

continued

Since
$$\left[\frac{\sin\,(n\Psi/2)}{\sin\,(\Psi/2)}\right]^2 = n + \sum_{k=1}^{n-1} 2(n-k)\cos\,(2k\Psi/2) \tag{6}$$

(5) can be expressed

$$\Omega_A = \frac{2}{n^2 d_\lambda} \int_0^{2\pi d_\lambda} [n + \sum_{k=1}^{n-1} 2(n-k)\cos\,(2k\Psi/2)]\; d\frac{\Psi}{2} \tag{7}$$

Integrating (7)

$$\Omega_A = \frac{2}{n^2 d_\lambda} \left[n\frac{\Psi}{2} + \sum_{k=1}^{n-1} \frac{2(n-k)}{2k} \sin\,(2k\,\frac{\Psi}{2}) \right]_0^{2\pi d_\lambda} \tag{8}$$

or

$$\Omega_A = \frac{2}{n^2 d_\lambda} \,[2\pi n d_\lambda + \sum_{k=1}^{n-1} \frac{n-k}{k} \sin\,(4\pi k d_\lambda)] \tag{9}$$

and
$$D = \frac{4\pi}{\Omega_A} = \frac{2\pi n^2 d_\lambda}{2\pi n d_\lambda + \sum_{k=1}^{n-1} \frac{n-k}{k} \sin\,(4\pi k d_\lambda)} \tag{10}$$

Therefore

$$D = \frac{n}{1 + (\lambda/2\pi n d) \sum_{k=1}^{n-1} \frac{n-k}{k} \sin\,(4\pi k d/\lambda)} \quad \text{q.e.d.} \tag{11}$$

We note that when $d = \lambda/4$, or a multiple thereof, the summation term is zero and $D = n$ exactly.

This problem and the next one are excellent examples of integration with change of variable and change of limits.

The final form for D in (11) above is well adapted for a computer program.

4-32 **Directivity of broadside array.**

The solution is similar to that for Prob. 4-31 with

$$\frac{\Psi}{2} = \pi d_\lambda \cos\theta$$

where $\theta = 0$, $\Psi/2 = \pi d_\lambda$ and when $\theta = \pi$, $\Psi/2 = -\pi d_\lambda$ so that (8) of Prob. 4-31 becomes

$$\Omega_A = \frac{2}{n^2 d_\lambda}\left[n\frac{\Psi}{2} + \sum_{k=1}^{n-1}\frac{(n-k)}{k}\sin\left(2k\frac{\Psi}{2}\right)\right]_{+\pi d_\lambda}^{-\pi d_\lambda}$$

$$\Omega_A = \frac{-2}{n^2 d_\lambda}\left[2\pi n d_\lambda + 2\sum_{k=1}^{n-1}\frac{n-k}{k}\sin(2\pi k d_\lambda)\right]$$

$$|\Omega_A| = \frac{4}{n^2 d_\lambda}\left[\pi n d_\lambda + \sum_{k=1}^{n-1}\frac{n-k}{k}\sin(2\pi k d_\lambda)\right]$$

$$D = \frac{4\pi}{\Omega_A} = \frac{\pi n^2 d_\lambda}{\pi n d_\lambda + \sum_{k=1}^{n-1}\frac{n-k}{k}\sin(2\pi k d_\lambda)}$$

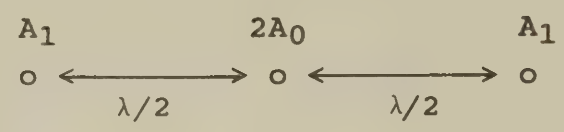
$$D = \frac{n}{1 + (\lambda/\pi n d)\sum_{k=1}^{n-1}\frac{n-k}{k}\sin(2\pi k d/\lambda)} \qquad \text{q.e.d.}$$

Note that when $d = \lambda/2$, or a multiple thereof, the summation term is zero and $D = n$ exactly.

See application of the above relations to the evaluation of D and of the main beam area Ω_A of an array of 16 point sources in Prob 11-34 (c) and (d).

*4-34 Three source array. Dolph-Tchebyscheff distribution.

Let the amplitudes (currents) of the 3 sources be as in the sketch



$d = \lambda/2$, $R = 10$

Let amplitude of center source $= 1 = 2A_0$

$n-1 = 2,\qquad T_2(x) = 2x^2 - 1 = R,$

$2x_o^2 - 1 = 10,\qquad 2x_o^2 = 11,$

$x_o^2 = 5.5,\qquad x_o = \pm\ 2.345$

$$E_3 = 2A_0 + 2A_1 \cos 2\frac{\psi}{2} = 2A_0 + 2A_1\left(2\cos^2\frac{\psi}{2} - 1\right)$$

$$E_3 = 2A_0 + 2A_1\ (2w^2 - 1)$$

Let $w = x/x_o$ so

$$E_3 = 2A_0 + 2A_1\left(2\frac{x^2}{x_o^2} - 1\right) = \frac{4A_1}{x_o^2}x^2 + 2A_o - 2A_1 = 2x^2 - 1$$

$$E_3 = 0.728\ A_1x^2 + (A_0 - 2A_1) = 2x^2 - 1$$

Therefore $0.728\ A_1 = 2$ and $A_1 = 2.75$

$2A_0 - 2A_1 = -1$ and $2A_0 = 5.5 - 1 = 4.5$

Thus, normalizing $2A_0 = 1$ and $A_1 = \dfrac{2.75}{4.5} = \boxed{0.61}$

*4-34 continued

Amplitude distribution is

0.61 1.00 0.61

Pattern has 4 minor lobes.

For center source, amplitude = 1.

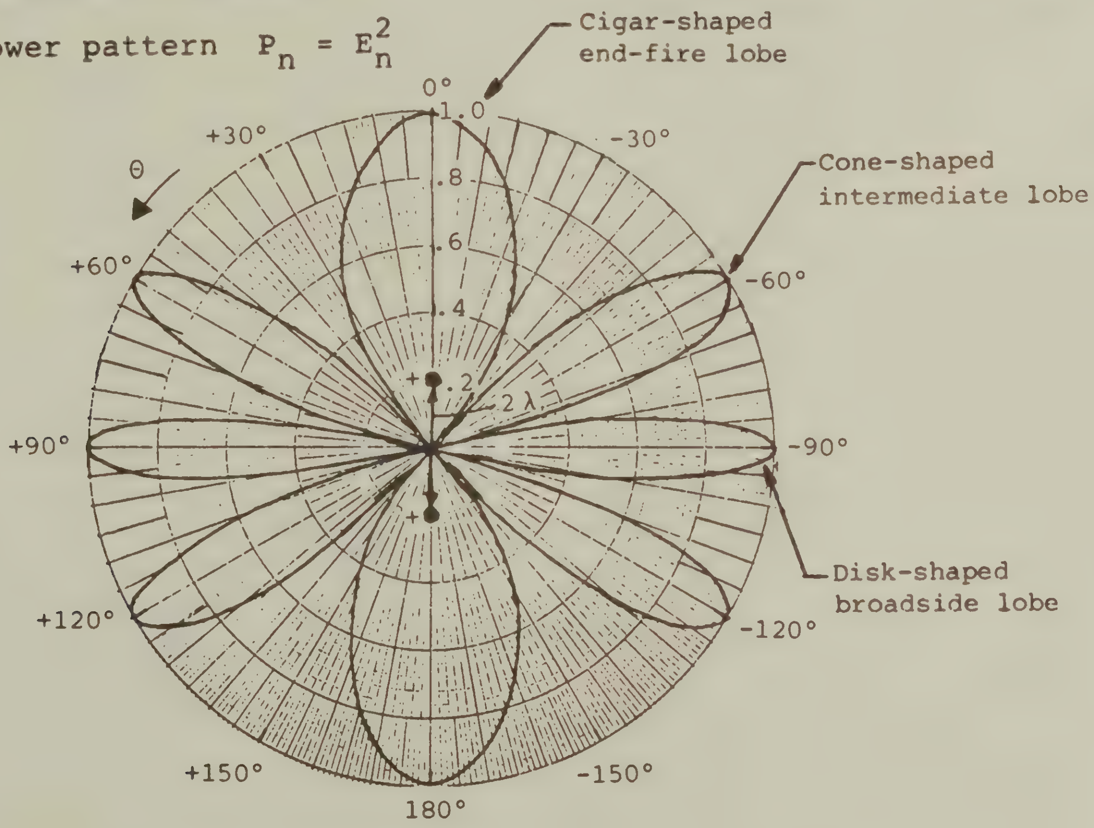
The side source amplitudes for different R values are:

R	A_1
8	0.64
10	0.61
12	0.59
15	0.57

---

*4-38 Two sources in phase.

(a) Power pattern $P_n = E_n^2$



continued

*4-38 continued

Instructional comment to pass on to students:

The lobes with narrowest beam widths are broadside (±90°), while the widest beam width lobes are endfire (0° and 180°). The four lobes between broadside and end-fire are intermediate in beam width. In three-dimensions the pattern is a figure-of-revolution around the array axis (0° and 180° axis) so that the broadside beam is a flat disk, the end-fire lobes are thick cigars while the intermediate lobes are cones. The accompanying figure is simply a cross section of the three-dimensional space figure.

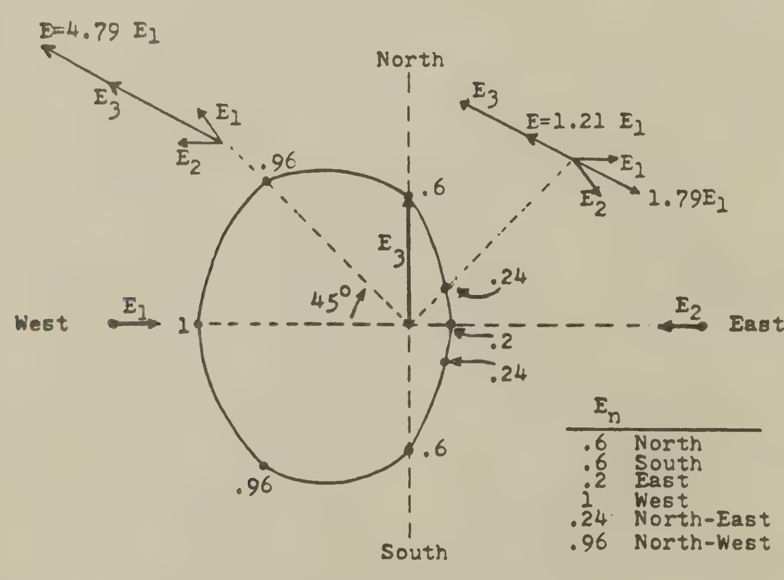
4-39 Two sources in opposite phase.

Maximum at: 0°, 180°, ± 70.5°, ± 109.5°

Nulls at: ± 48.2°, ± 90°, ± 131.8°

4-41 Three unequal sources.

Phasor addition



4-45 Stray factor and directive gain.

$$\text{Stray factor} = \frac{\Omega_m}{\Omega_A}$$

where Ω_A = total beam area

Ω_M = main lobe beam area

Ω_m = minor lobe beam area

$$\frac{\Omega_m}{\Omega_A} = \frac{\iint_{4\pi-\Omega_M} P_n(\theta,\phi)\, d\Omega}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega}$$

Average directive gain over minor lobes $= DG_{av}(\text{minor}) = \dfrac{1}{4\pi-\Omega_M} \displaystyle\iint_{4\pi-\Omega_M} DP_n(\theta,\phi)\, d\Omega$

where $D = 4\pi/\Omega_A$

Therefore $\quad DG_{av}(\text{minor}) = \dfrac{1}{4\pi-\Omega_M} \dfrac{\displaystyle\iint_{4\pi-\Omega_M} 4\pi P_n(\theta,\phi)\, d\Omega}{\Omega_A}$

$$= \frac{4\pi}{4\pi-\Omega_M} \frac{\Omega_m}{\Omega_A}$$

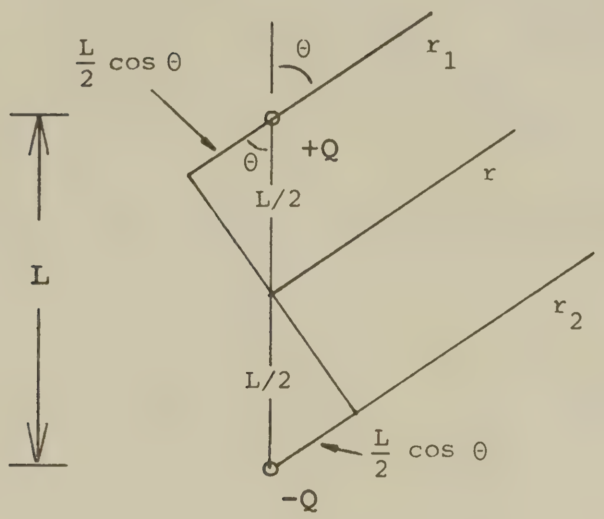
If $\Omega_M << 4\pi$ (antenna highly directive),

$DG_{av}(\text{minor}) \cong \dfrac{\Omega_m}{\Omega_A}$ (stray factor) q.e.d.

CHAPTER 5

THE ELECTRIC DIPOLE AND THIN LINEAR ANTENNAS

*<u>5-1</u> Electric dipole. Superposition.



(a) $$V(\text{at } r) = \frac{Q}{4\pi\epsilon r_1} - \frac{Q}{4\pi\epsilon r_2}$$

$$r_1 = r - (L/2)\cos\theta$$

$$r_2 = r + (L/2)\cos\theta$$

$$V = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r - (L/2)\cos\theta} - \frac{1}{r + (L/2)\cos\theta}\right]$$

$$V = \frac{Q}{4\pi\epsilon}\left[\frac{r + (L/2)\cos\theta - r + (L/2)\cos\theta}{r^2 + (L/2)^2\cos^2\theta}\right]$$

For r>>L, $$V = \frac{QL\cos\theta}{4\pi\epsilon r^2}$$ q.e.d.

(b) $$E = -\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial V}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}$$

*<u>5-1</u> continued

$$= \frac{-QL}{4\pi\epsilon}\left[-\hat{\mathbf{r}}\frac{2\cos\theta}{r^3} - \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\sin\theta}{r^2} + \hat{\boldsymbol{\phi}}\,0\right]$$

$$= \hat{\mathbf{r}}\frac{QL\cos\theta}{2\pi\epsilon r^3} + \hat{\boldsymbol{\theta}}\frac{QL\sin\theta}{4\pi\epsilon r^3}$$

or

$$E_r = \boxed{\frac{QL\cos\theta}{2\pi\epsilon r^3}}, \quad E_\theta = \boxed{\frac{QL\sin\theta}{4\pi\epsilon r^3}} \quad \text{and } E_\phi = \boxed{0}$$

<u>5-2</u> 2 λ antenna.

(a) $E_n(\theta) = \boxed{\frac{\cos(2\pi\cos\theta) - 1}{\sin\theta}}$

(b) R (at I_{max}) = $\boxed{259\ \Omega}$

(c) R (at terminals) = $\boxed{\infty\ \Omega}$

(d) R (at λ/8 from I_{max}) = $\boxed{518\ \Omega}$

---

*<u>5-3</u> λ/2 antenna.

(a) $E_n(\theta) = \boxed{\tan\theta \sin[(\pi/2)\cos\theta]}$

(b) R = $\boxed{168\ \Omega}$

(c) R [from (b)] = $\boxed{168\ \Omega}$

R (sinusoidal I) = $\boxed{73\ \Omega}$

R (short dipole) = $\boxed{197\ \Omega}$

continued

*5-3 continued

(d) 168 Ω is appropriate for uniform current.

73 Ω is appropriate for sinusoidal current.

197 Ω assumes uniform current, but the short dipole formula does not take into account the difference in distance to different parts of the dipole (assumes $\lambda >> L$) which is not appropriate and leads to a larger resistance (197 Ω) as compared to the correct value of 168 Ω.

5-4 $\lambda/2$ antennas in echelon.

$$E_n(\theta) = \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \cos\left[\frac{\sqrt{2}\,\pi}{4} \cos[(\pi/4) + \theta]\right]$$

---

*5-6 1λ and 10λ antennas with traveling waves.

(a) From (5-8-15),

$$E_n(\phi) = \frac{\sin\phi}{1 - p\cos\phi} \left[\sin\pi\left(\frac{1}{p} - \cos\phi\right)\right]$$

Patterns have 4 lobes.

(b) Pattern has 40 lobes.

---

*5-7 Isotropic antenna. Radiation resistance.

$$E = \frac{10\,I}{r} \quad \text{so} \quad S = E^2/Z = \frac{100 I^2}{r^2 Z}$$

Let P = power over sphere = $4\pi r^2 S$, which must equal power I^2R to the antenna terminals. Therefore, $I^2R = 4\pi r^2 S$ and

$$R = \frac{1}{I^2}\, 4\pi r^2 \, \frac{100\, I^2}{r^2\, 120\pi} = \frac{400}{120} = \boxed{3.33\ \Omega}$$

5-8 Short dipole. Directivity. Resistance.

(a) $E_n(\theta) = \sin\theta$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int\int_{4\pi} \sin^2\theta \, d\Omega} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \sin^2\theta \sin\theta \, d\theta d\Omega}$$

$$= \frac{4\pi}{2\pi \int_0^{\pi} \sin^3\theta \, d\theta} = \frac{4\pi}{2\pi \frac{4}{3}} = \frac{3}{2} = \boxed{1.5 \text{ or } 1.76 \text{ dBi}}$$

(d) From (a), $\Omega_A = 8\pi/3 = \boxed{8.38 \text{ sr}}$

(e) From (5-3-14),

$$R_r = 790 \left[\frac{I_{av}}{I_o}\right]^2 L_\lambda^2 = 790 \left[\frac{1}{2}\right]^2 \left[\frac{1}{15}\right]^2 = \boxed{0.878\ \Omega}$$

(b) $G = kD = \dfrac{0.878}{0.878 + 1} \times 1.5 = \boxed{0.70 \text{ or } -1.54 \text{ dBi}}$

(c) $A_e = kA_{em}$ where $A_{em} = \lambda^2/\Omega_A = \dfrac{3}{8\pi}\lambda^2$

Therefore $A_e = \dfrac{0.878}{0.878 + 1} \times \dfrac{3}{8\pi} = \boxed{0.058\ \lambda^2}$

***5-9** Conical pattern. Beam area.

(a) $$\Omega_A = \int_0^{360^\circ}\int_0^{60^\circ} d\Omega = 2\pi \int_0^{60^\circ} \sin\theta \, d\theta = -2\pi \cos\theta \Big|_0^{60^\circ}$$

$$= \boxed{\pi \text{ sr}}$$

continued

*5-9 continued

(b) $D = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{\pi} = \boxed{4}$

5-10 Conical pattern.

(a) $\Omega_A = 2\pi \int_0^{45^\circ} \sin\theta \, d\theta = \boxed{1.84 \text{ sr}}$

(b) $D = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{1.84} = \boxed{6.83}$

(c) $A_e = A_{em} = \lambda^2/\Omega_A = \lambda^2/1.84 = \boxed{0.543\ \lambda^2}$

(d) $I^2R_r = \Omega_A r^2 \dfrac{E^2}{Z}$

$R_r = \dfrac{1}{2^2}\, 1.84 \times 50^2\, \dfrac{5^2}{377} = \boxed{76.3\ \Omega}$

---

*5-11 Directional pattern in θ and ϕ.

(a)

$\Omega_A = \int_0^{120^\circ} \int_{45^\circ}^{90^\circ} \sin\theta \, d\theta d\phi = -\dfrac{2\pi}{3} \cos\theta \Big|_{45^\circ}^{90^\circ}$

$= \boxed{1.48 \text{ sr}}$

(c) $R_r = \dfrac{1}{I^2}\, \Omega_A r^2 \dfrac{E^2}{Z} = \dfrac{1}{5^2}\, 1.48 \times 500^2\, \dfrac{3^2}{377}$

$= \boxed{354\ \Omega}$

5-12 Directional pattern in θ and φ.

(a)

$$\Omega_A = \int_0^{90^\circ} \int_{30^\circ}^{60^\circ} \sin\theta \, d\theta d\phi = -\frac{\pi}{2} \cos\theta \Big|_{30^\circ}^{60^\circ} = \boxed{0.575 \text{ sr}}$$

$$D = \frac{4\pi}{0.575} = \boxed{21.9}$$

(b) $$A_e = A_{em} = \lambda^2/\Omega_A = \frac{\lambda^2}{0.575} = \boxed{1.74\ \lambda^2}$$

(c) $$R_r = \frac{1}{I^2} \Omega_A r^2 \frac{E^2}{Z} = \frac{1}{3^2} 0.575 \times 100^2 \frac{2^2}{377} = \boxed{6.78\ \Omega}$$

---

*5-13 Directional pattern with back lobe.

(a)

$$\Omega_A = 2\pi \int_0^{30^\circ} \sin\theta \, d\theta + \frac{2\pi}{3^2} \int_{90^\circ}^{180^\circ} \sin\theta \, d\theta$$

$$= 2\pi\ (0.134 + 0.111) = 2\pi \times 0.245$$

$$D = \frac{4\pi}{2\pi \times 0.245} = \boxed{8.16}$$

(b)

$$R_r = \frac{1}{I^2} \Omega_A r^2 \frac{E^2}{Z} = \frac{1}{4^2} 2\pi \times 0.245 \times 200^2 \frac{8^2}{120\pi}$$

$$= \boxed{653\ \Omega}$$

5-14 Short dipole.

The current I given in the problem is a peak value, so we put

$$\underset{\text{Power input}}{\frac{1}{2} I^2 R_r} = \underset{\text{Power radiated}}{\iint_{4\pi} S r^2 \, d\Omega}$$

continued

*<u>5-14</u> continued

where $S = \dfrac{E^2}{Z}$ and E is as given

so $$R_r = \frac{2}{I^2} 2\pi \frac{30^2 \beta^2 \ell^2 I^2}{r^2\ 120\pi} r^2 \int_0^{\pi} \sin^3 \theta\, d\theta$$

$$= 80\pi^2 (\ell/\lambda)^2 = \boxed{790\ (\ell/\lambda)^2\ \Omega}$$

<u>5-15</u> Equivalence of pattern factors.

(1) $$\text{Field pattern} = \frac{\sin \frac{n\Psi}{2}}{\sin \frac{\Psi}{2}} \qquad (4\text{-}6\text{-}8)$$

where $\Psi = \beta d \cos \phi + \delta$

(2) $$\text{Field pattern} = \sin \phi \frac{\sin\left[\frac{\omega b}{2pc}(1 - p\cos \phi)\right]}{1 - p\cos \phi} \qquad (5\text{-}8\text{-}15)$$

For ordinary end-fire $\Psi = \beta d(\cos \phi - 1)$.

Also if d is small (1) becomes

$$\frac{\sin\left[\frac{\beta nd}{2} (1 - \cos \phi)\right]}{\frac{\beta d}{2} (1 - \cos \phi)}$$

For large n, nd $\cong$ b. Also multiplying by the source factor $\sin \phi$ and taking the constant $\beta d/2 = 1$ in the denominator, (1) becomes

$$\sin \phi \frac{\sin\left[\frac{\beta d}{2} (1 - \cos \phi)\right]}{(1 - \cos \phi)}$$

5-15 continued

which is the same as (2) for p = 1

since $$\frac{\omega b}{2pc} = \frac{2\pi fb}{2f\lambda} = \frac{\beta b}{2} \qquad \text{q.e.d.}$$

Note that for a given length b, the number n is assumed to be sufficiently large that d can be small enough to allow $\sin \Psi/2$ in (1) to be replaced by $\Psi/2$.

5-16 Relation of radiation resistance to beam area.

Taking I as the rms value we set

$$\underset{\text{Power input}}{I^2 R_r} = \underset{\text{Power radiated}}{S r^2 \Omega_A}$$

Therefore $$R_r = \frac{S r^2}{I^2} \Omega_A \qquad \text{q.e.d.}$$

5-17 Cross-field.

Hint: Find locations in the near field where $|E_r| = |E_\theta|$ and where E_r and E_θ are in time-phase quadrature. There is one location in each quadrant.

Heinrich Hertz published diagrams showing the locations in 1889. See vol. 36 of Wiedemann's Annalen, 1889.

CHAPTER 6

THE LOOP ANTENNA

6-1 The $3\lambda/4$ diameter loop.

$$C_\lambda = \pi \frac{3}{4} = 2.36$$

From (6-5-8) or Table 6-2, the E_ϕ pattern is given by

$$J_1(C_\lambda \sin \theta)$$

See Figure 6-9.

---

*6-2 The 1λ square loop.

Pattern is that of 2 point sources in opposite phase.

Referring to Section 4-2b we have for $\frac{d_r}{2} = 2\pi \frac{\lambda}{2} = \pi$,

$$E_n(\phi) = \sin (\pi \cos \phi)$$

resulting in a 4-lobed pattern with maxima at $\phi = \pm 60^\circ$ and $\pm 120^\circ$ and nulls at 0°, $\pm 90^\circ$ and 180°.

6-3 The $\lambda/10$ diameter loop antenna.

Ω_A is the same as for a short dipole ($= 8\pi/3$ sr).

See Prob. 5-8a.

Therefore,

$$A_{em} = \lambda^2/\Omega_A = (3/8\pi)\lambda^2 = \boxed{0.119\ \lambda^2}$$

***6-4** Radiation resistance of loop. Bessel functions.

From (6-8-13) for loop of any size

$$R_r = 60\pi^2 C_\lambda \int_0^{2C_\lambda} J_2(y)\, dy$$

where $$C_\lambda = \pi \frac{3}{4} = 2.36$$

$$2C_\lambda = 4.71$$

From (6-8-16)

$$\int_0^{2C_\lambda} J_2(y)\, dy = \int_0^{2C_\lambda} J_o(y)\, dy - 2J_1(2C_\lambda)$$

By integration of the $J_o(y)$ curve from 0 to $2C_\lambda$ (=4.71)

$$\int_0^{2C_\lambda} J_o(y)\, dy = 0.792$$

From tables (Jahnke and Emde)

$$J_1(2C_\lambda) = J_1(4.71) = -0.2816$$

and $$\int_0^{2C_\lambda} J_2(y)\, dy = 0.7920 + 2 \times 0.2816 = 1.355$$

Therefore $$R_r = 60\ \pi^2\ 2.36 \times 1.355 = \boxed{1894\ \Omega}$$

Round off to 1890 Ω

6-5 Pattern, radiation resistance and directivity of loops.

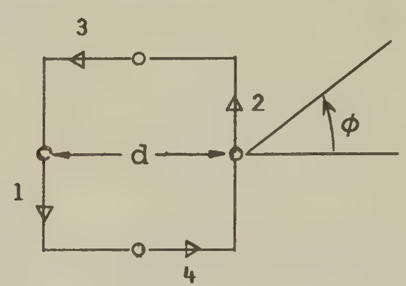
Since all of the loops have $C_\lambda > 1/3$, the general expression for E_ϕ in Table 6-2 must be used.

From Table 6-2 and Figures 6-12 and 6-13, the radiation resistance and directivity values are:

continued

Diameter	C_λ	R_r	Directivity
$\lambda/4$	0.785	76 Ω	1.5
1.5λ	4.71	2340 Ω	3.82
8λ	25.1	14800 Ω	17.1

6-6 Small square loop.



The field pattern E(1,2) of sides 1 and 2 of the small square loop is the product of the pattern of 2 point sources in opposite phase separated by d as given by

$$\sin\,[(d_r/2)\cos\phi]$$

and the pattern of a short dipole as given by

$$\cos\phi$$

or $$E(1,2) = \cos\phi\,\sin\,[(d_r/2)\cos\phi]$$

For small d this reduces to

$$E_n(1,2) = \cos^2\phi$$

6-6 continued

The pattern of sides 3 and 4 is the same rotated through 90° or in terms of ϕ is given by

$$E_n(3,4) = \sin^2\phi$$

The total pattern in the plane of the square loop is then

$$E_n(\phi) = E_n(1,2) + E_n(3,4)$$

$$= \cos^2\phi + \sin^2\phi = 1$$

Therefore $E(\phi)$ is a constant as a function of ϕ and the pattern is a circle. q.e.d.

---

*6-7 Circular loop.

See Probs. 6-1 and 6-5.

Radiation resistance and directivity values are:

Diameter	C_λ	R_r	Directivity
$\lambda/3$	1.05	180 Ω	1.5
0.75λ	2.36	1550 Ω	1.2
2λ	6.28	4100 Ω	3.6

6-8 Small loop resistance.

(a) $$R_r = \frac{S\, r^2 \Omega_A}{I^2} = \frac{E_{max}^2\, r^2 \Omega_A}{Z\, I^2}$$

From (6-7-2) and Table 6-2,

$$|E_\phi| = \frac{120\,\pi^2 I\, A}{r\ \lambda^2} \sin\theta = E_{max} \sin\theta$$

continued

6-8 continued

$$\Omega_A = 2\pi \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta = 2\pi \frac{4}{3} = \frac{8}{3}\pi$$

Therefore,

$$R_r = \frac{120^2 \pi^4 I^2 A^2}{r^2 \lambda^4} \quad \frac{r^2}{120\,\pi\, I^2} \quad \frac{8\pi}{3}$$

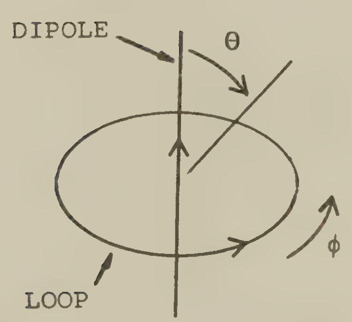
$$= 320\,\pi^4 (A/\lambda^2)^2\ \Omega \quad \text{q.e.d.}$$

$$= 197\ C_\lambda^4\ \Omega$$

(b) $\Omega_A = 4\pi$, $A_e = \lambda^2/\Omega_A = \lambda^2/4\pi$ q.e.d.

<u>6-9</u> Loop and dipole for circular polarization.

Uniform currents are assumed.



$$E_\phi(\theta)(\text{loop}) = \frac{120\pi^2\ I\ A \sin\theta}{r\ \lambda^2} \qquad (1)$$

$$E_\theta(\theta)(\text{dipole}) = \frac{j\ 60\pi\ I\ L \sin\theta}{r\ \lambda} \qquad (2)$$

$$R_r(\text{loop}) = 320\pi^4 (A/\lambda^2)^2\Omega \qquad (3)$$

$$R_r(\text{dipole}) = 80\pi^2 L_\lambda^2 \qquad (4)$$

For equal power inputs

$$I_{\text{loop}}^2 R_r(\text{loop}) = I_{\text{dipole}}^2\ R_r(\text{dipole})$$

$$\frac{I^2_{loop}}{I^2_{dipole}} = \frac{R_r(\text{dipole})}{R_r(\text{loop})} = \frac{80\pi^2 L^2_\lambda}{320\pi^4 (A/\lambda^2)^2} \tag{5}$$

$$= \frac{L^2_\lambda}{4\pi^2 (A/\lambda^2)^2} \tag{6}$$

$$\frac{I_{loop}}{I_{dipole}} = \frac{L_\lambda}{2\pi\ (A/\lambda^2)} \tag{7}$$

Therefore

$$E_\phi(\theta)(\text{loop}) = \frac{120\pi^2 L_\lambda\ I_{dipole} A \sin\theta}{r\ \lambda^2 2\pi (A/\lambda^2)}$$

$$= \frac{60\pi I_{dipole} L \sin\theta}{r\lambda} \tag{8}$$

which is equal in magnitude to $E_\theta(\theta)$ (dipole) but in time-phase quadrature (no j).

Since the 2 linearly-polarized fields (E_ϕ of the loop and E_θ of the dipole) are at right angles, are equal in magnitude and are in time-phase quadrature, the total field of the loop-dipole combination is everywhere circularly polarized with a sin θ pattern. q.e.d.

Equating the magnitude of (1) and (2) (fields equal and currents equal) we obtain

$$\frac{L}{\lambda} = 2\pi \frac{A}{\lambda^2} \tag{9}$$

which satisfies (7) for equal loop and dipole currents. Thus, (9) is a condition for circular polarization.

Substituting $A = \pi \dfrac{d^2}{4}$

continued

6-9 continued

where d = loop diameter in (9) and putting $C = \pi d$,

$$\frac{L}{\lambda} = 2\pi \frac{\pi d^2}{4\lambda^2} = \frac{1}{2}\frac{C^2}{\lambda^2} \tag{10}$$

we obtain

$$C_\lambda = (2L_\lambda)^{\frac{1}{2}} \tag{11}$$

as another expression of the condition for circular polarization.

Thus, for a short dipole $\lambda/10$ long, the loop circumference must be

$$C_\lambda = (2 \times 0.1)^{\frac{1}{2}} = 0.45 \tag{12}$$

and the loop diameter

$$d = \frac{0.45\,\lambda}{\pi} = 0.14\lambda$$

or 1.4 times the dipole length. If the dipole current tapers to zero at the ends of the dipole, the condition for CP is

$$\frac{L}{\lambda} = 4\pi \frac{A}{\lambda^2} \tag{13}$$

and

$$C_\lambda = (L_\lambda)^{\frac{1}{2}} \tag{14}$$

For a $\lambda/10$ dipole the circumference must now be

$$C_\lambda = (0.1\,)^{\frac{1}{2}} = 0.316$$

and the loop diameter

$$d = \frac{0.316\lambda}{\pi} \cong 0.1\lambda$$

or approximately the same as the dipole length.

The condition of (11) is applied in the Wheeler-type helical antenna. See Section 7-19, equation (7-19-4) and Prob. 7-6.

CHAPTER 7

THE HELICAL ANTENNA

*7-1 An 8-turn helix.

(a) The relative phase velocity for in-phase fields is given by (7-5-9) as

$$p = \frac{1}{\sin\alpha + \dfrac{\cos\alpha}{C_\lambda}}$$

The relative phase velocity for increased directivity is given by (7-5-12)

$$p = \frac{L_\lambda}{S_\lambda + \dfrac{2n+1}{2n}}$$

From the given value of frequency and diameter D, C_λ can be determined. Introducing it and the given values of α and n

$p = 0.802$ for in-phase fields

$p = 0.763$ for increased directivity

7-2 A 10-turn helix.

(a) See below.

$$\lambda = \frac{3 \times 10^8}{10^9} = 0.3 \text{ m} \qquad C = \pi \times 0.1 = 0.314$$

$$C_\lambda = \frac{0.314}{0.3} = 1.047 \qquad S_\lambda = \frac{0.07}{0.3} = 0.233$$

continued

(b) From (7-4-4)

$$\text{HPBW} \cong \frac{52^\circ}{C_\lambda (nS_\lambda)^{1/2}} = \frac{52^\circ}{1.047(10 \times 0.233)^{1/2}} = \boxed{32.5^\circ}$$

(c) From (7-4-7)

$$D \cong 12\ C_\lambda^2\ nS_\lambda = \boxed{30.7 \text{ or } 14.9 \text{ dBi}}$$

If losses are negligible the gain = D.

(d) Polarization is $\boxed{\text{RCP}}$.

(e) At 300 MHz, $\lambda = 3 \times 10^8/300 \times 10^6 = 1$ m

$C_\lambda = 0.314/1 = 0.314$. This is too small for the axial mode which requires that $0.7<C_\lambda<1.4$.

From (2-9-4)

$$D = \frac{41\ 000}{32.5^2} = 38.8 \text{ or } 15.9 \text{ dBi}$$

or 1 dB higher. The lower value is more realistic.

(a) The pattern from (7-7-3) is given by

$$E_n(\phi) = \sin(90^\circ/n) \frac{\sin(n\Psi/2)}{\sin \frac{\Psi}{2}} \cos\phi \qquad (1)$$

where $\Psi = 360^\circ[S_\lambda(1 - \cos\phi) + (1/2n)]$

Calculation of the pattern is facilitated by using a computer. Thus, the first 2 factors of (1) are calculated by the BASIC program in Appendix B-2,

7-2 continued

where in line 10:

N = number of turns

D = spacing = S_λ

S = phase shift between turns = $2\pi\ S_\lambda + (\pi/N)$

MF = multiplying or normalizing factor
= 67 sin $(\pi/2N)$

The factor cos ϕ is also required to account for the single turn pattern but this has only a small effect on the main lobe for long helices. This factor may by included in the calculation by changing line 80 to read:

R = MF*ABS(R)*CA

7-3 A 30-turn helix.

(a) From (7-4-4)

$$\text{HPBW} \cong \frac{52^\circ}{C_\lambda (nS_\lambda)^{1/2}} = \frac{52^\circ}{\frac{\pi}{3}\ (30 \times 0.2)^{1/2}} = \boxed{20.3^\circ}$$

(b) For zero losses, G = D

From (7-4-7)

$$D \cong 12\ C_\lambda^2\ nS_\lambda = 12\ (\pi/3)^2\ 30 \times 0.2 = \boxed{79 \text{ or } 19 \text{ dBi}}$$

(c) $\boxed{\text{RCP}}$

See note in text (p. 339) and also Prob. 7-2 above about using the BASIC program in Appendix B-2 to calculate the pattern.

The end-fire array with increased directivity in Program 4, Appendix B-2 has 24 sources with 0.25_λ spacing so $nS_\lambda = 24 \times 0.25 = 6$ which is the same length as for the above 30-turn helix with 0.2_λ spacing ($30 \times 0.2 = 6$).

continued

7-3 continued

Therefore, the main lobe pattern in Fig. B-1-4 is nearly the same as for the above 30-turn helix, the single-turn pattern factor (=cosϕ) having only a small effect on the main lobe. The HPBW of the pattern in Fig. B-2-4 is about 23°.

7-4 Helices, left and right.

Assuming that x is horizontal,

(a) LHP

(b) LVP

---

*7-6 Normal-mode helix.

See solution to Prob. 6-9.

(a) $\boxed{D_\lambda = (2\ H_\lambda)^{1/2}/\pi}$

(b) $\boxed{E = \sin\ \theta}$

CHAPTER 8

THE BICONICAL ANTENNA AND ITS IMPEDANCE

*8-3 The 2° cone.

From (8-2-21) for Z_k, (8-4-4) and (8-4-5) for Z_m and (8-4-6) for Z_i,

$$Z_i = \boxed{270 + j350\ \Omega}$$

8-4 Bow-tie antenna.

From Fig. 8-17

$$G = D = \boxed{3\text{ dBi}}$$

From Fig. 8-15

$$Z = \boxed{200 - j60\ \Omega}$$

8-5 Monotriangular antenna.

From Fig. 8-15

$$Z = \boxed{100 - j20\ \Omega}$$

8-6 Monoconical antenna.

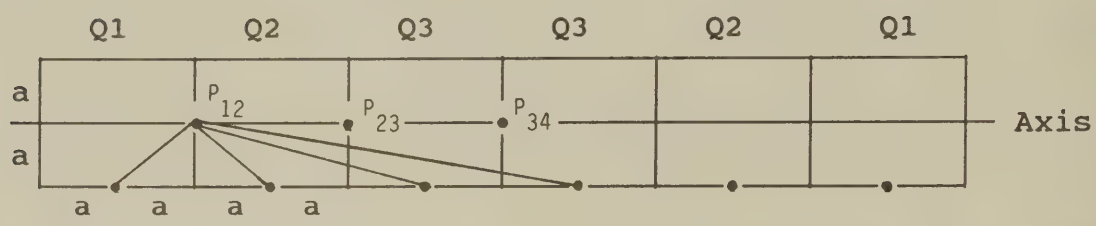
From Fig. 8-15

$$Z = \boxed{53 + j0\ \Omega}$$

CHAPTER 9

THE CYLINDRICAL ANTENNA; THE MOMENT METHOD

9-2 Charge distribution. Moment method.



Divide rod into 6 equal segments. By symmetry charges are as shown. Neglecting end faces, the potential at point P_{12} is given by

$$V(P_{12}) = \frac{1}{4\pi\epsilon}\left[\frac{Q_1}{\sqrt{2}\,a} + \frac{Q_2}{\sqrt{2}\,a} + \frac{Q_3}{\sqrt{10}\,a} + \frac{Q_3}{\sqrt{26}\,a} + \frac{Q_2}{\sqrt{50}\,a} + \frac{Q_1}{\sqrt{82}\,a}\right]$$

Writing similar expressions for $V(P_{23})$ and $V(P_{34})$, equating them (since the potential is constant along the rod) and solving for the charges yields:

$$Q_1 : Q_2 : Q_3 = \boxed{1.582 : 1.062 : 1.000}$$

9-6 $\lambda/10$ dipole impedance. Convergence.

Using Z_s values of Table 9-4, plot R_s vs. N to suitable large scale and suppressed zero and note that R_s approaches a constant (convergence) value as N becomes large which should agree with Richmond's value given following (9-17-34). Calculate R_s for larger values of N than 7, if desired. Do same for X_s.

CHAPTER 10

SELF AND MUTUAL IMPEDANCES

*<u>10-1</u> A $5\lambda/2$ Antenna. Sine and cosine integrals.

From (10-3-49)

$$Z_{11} = 30\ [\ 0.577 + \ln\ (2\pi n) - \mathrm{Ci}\ (2\pi n) + j\ \mathrm{Si}\ (2\pi n)]$$

where $n = 5$

Since $2\pi n = 2\pi 5 = 10\pi >> 1$, we have from (5-6-18)

$$\mathrm{Ci}\ (10\pi) = \frac{\sin\ (10\pi)}{10\pi} = 0$$

and from (5-6-22)

$$\mathrm{Si}\ (10\pi) = \frac{\pi}{2} - \frac{\cos\ (10\pi)}{10\pi} = \frac{\pi}{2} - \frac{1}{10\pi} = 1.539$$

and $R_{11} = 30\ [0.577 + \ln\ (10\pi) - 0] = \boxed{120.7\ \Omega}$

$X_{11} = 30 \times 1.539 = \boxed{46.2\ \Omega}$

<u>10-2</u> Parallel side-by-side $\lambda/2$ antennas. Sine and cosine integrals.

From (10-5-6)

$$R_{21} = 30\ \{2\mathrm{Ci}(\beta d) - \mathrm{Ci}[\beta(\sqrt{d^2 + L^2} + L)] - \mathrm{Ci}[\beta(\sqrt{d^2 + L^2} - L)]\}$$

where $d = 0.15\lambda$

$L = 0.5\lambda$

and

$$R_{21} = 30\{2\mathrm{Ci}(0.942) - \mathrm{Ci}(6.42) - \mathrm{Ci}(0.138)\}$$

continued

10-2 continued

From (5-6-16) or Ci table or from Fig. 5-12a and from (5-6-18) we have

$$R_{21} = 30\{0.60 - 0 - 0.577 - \ln 0.138\}$$

$$= \boxed{60.1\ \Omega} \quad \text{compare with Fig. 10-12.}$$

From (10-5-8)

$$X_{21} = -30\{2\mathrm{Si}(\beta d) - \mathrm{Si}[\beta(\sqrt{d^2 + L^2} + L)] - \mathrm{Si}[\beta(\sqrt{d^2 + L^2} - L)]\}$$

and

$$X_{21} = -30\{2\mathrm{Si}(0.942) - \mathrm{Si}(6.42) - \mathrm{Si}(0.138)\}$$

From (5-6-20) or Si table or from Fig. 5-12b and from (5-6-21) we have

$$X_{11} = -30\{1.8 - 1.42 - 0.138\}$$

$$= \boxed{-7.3\ \Omega} \quad \text{compare with Fig. 10-12.}$$

10-3 Two $\lambda/2$ antennas in echelon. Sine and cosine integrals.

Use (10-7-1), (10-7-2), (5-6-16) and (5-6-20)

where $h = 1.25\lambda$

$d = 0.25\lambda$

---

*10-5 Three side-by-side antennas.

$$R_a = R_s - R_{ab} + R_{ac} = 100 - 40 - 10 = \boxed{50\ \Omega}$$

$$R_b = R_s - 2R_{ab} = 100 - 80 = \boxed{20\ \Omega}$$

$$R_c = R_a = \boxed{50\ \Omega}$$

CHAPTER 11

ARRAYS OF DIPOLES AND OF APERTURES

*<u>11-1</u> **Two λ/2-element broadside array.**

(a) From (11-2-27),

$$G_f(\phi)\ (A/HW) = [2R_{00}/(R_{11}+R_{12})]^{\frac{1}{2}} \cos[(d_r \cos\phi)/2]$$

In broadside direction $\phi = \pi/2$

so $$G_f(\phi)\ (A/HW) = [2R_{00}/(R_{11}+R_{12})]^{\frac{1}{2}}$$

where $R_{00} = R_{11} = 73.1\ \Omega$

and R_{12} is a function of the spacing as given in Table 10-1 (p. 427). A few values of the gain for spacings from 0 to 1λ are listed below:

Spacing λ	Gain over λ/2 reference
0.0	1.00
0.1	1.02
0.2	1.08
0.3	1.19
0.4	1.36
0.5	1.56
0.6	1.72
0.7	1.74
0.8	1.64
0.9	1.49
1.0	1.38
etc.	

(b) By interpolation, the highest gain occurs for a spacing of about 0.67λ for which the gain is about 1.76 or 4.9 dB (=7.1 dBi). At spacings over 1λ no gains exceed this.

continued

*11-1 continued

Note that

$$D(\lambda/2) = 4\pi\ A_{em}(\lambda/2)/\lambda^2 = 4\pi\ (30/73.1\pi) = 1.64 \text{ or } 2.15 \text{ dBi}$$

so D of 2 in-phase $\lambda/2$ elements at 0.67λ spacing is equal to 4.9 + 2.15 = 7.1 dBi as above.

11-2 **Two $\lambda/2$ element end-fire array.**

(a) From (11-3-18),

$$G_f(\phi)(A/HW) = [2R_{00}/(R_{11} - R_{12})]^{\frac{1}{2}} \sin [(d_r \cos \phi)/2]$$

When $d = \lambda/2$,

$$G_f(\phi)(A/HW) = [2R_{00}/(R_{11} - R_{12})]^{\frac{1}{2}} \sin [(\pi/2) \cos \phi]$$

where $R_{00} = R_{11} = 73.1\ \Omega$

and from Table 10-1, $R_{12} = -12.7\ \Omega$

so $G_f(\phi)(A/HW) = 1.31 \sin [(\pi/2) \cos \phi]$

For unit gain,

$$\sin [(\pi/2) \cos \phi] = 1/1.31 = 0.763$$

or $\cos \phi = 49.8^\circ/90^\circ = 0.553$

and $\phi = \boxed{\pm 56^\circ,\ \pm 124^\circ}$

(b) When $\lambda/4$,

$$G_f(\phi)(A/HW) = [2R_{00}/(R_{11} - R_{12})]^{\frac{1}{2}} \sin [(\pi/4) \cos \phi]$$

where $R_{00} = R_{11} = 73.1\ \Omega$

and from Table 10-1, $R_{12} = 40.9\ \Omega$

11-2 continued

so $G_f(\phi)(A/HW) = 2.13 \sin [(\pi/4)\cos \phi]$

For unit gain,

$$\sin [(\pi/4)\cos \phi] = 1/2.13 = 0.47$$

$\cos \phi = 28^\circ/45^\circ = 0.62$ and $\phi = \boxed{\pm 52^\circ, \pm 128^\circ}$

11-6 Two-element array with unequal currents.

(a) $$G_f(\theta) = \{R_{11}/[R_{11}(1+a^2) + 2aR_{12}\cos \delta]\}^{\frac{1}{2}}$$

$$\times (1 + a^2 + 2a \cos \Psi)^{\frac{1}{2}}$$

where $\Psi = d_r \sin \theta + \delta$

$G_f(\phi) = G_f(\theta)$ but with $\Psi = d_r \cos \phi + \delta$

---

*11-7 Impedance of D-T array.

(a) From Prob. 4-15 the 6 sources have the distribution:

1	2	3	4	5	6
0.93	0.84	1.00	1.00	0.84	0.93

Normalizing the current for element 1, the distribution is

1.00	0.90	1.08	1.08	0.90	1.00

Using impedance data from Chap. 10 and assuming thin elements, the driving point impedance of element 1 is

continued

*<u>11-7</u> continued

$$R_1 = 73 + j43 + 0.9\,(-12 - j29) + 1.08(3 + j18)$$
$$+1.08(-2 - j12) + 0.9(1 + j10) - 1 - j3$$
$$= 73 - 10.8 + 3.2 - 2 + 0.9 - 1$$
$$+j(43 - 26 + 19.4 - 13 + 9 - 3)$$
$$= \boxed{63 + j29\ \Omega} = R_6$$

In like manner,

$$R_2 = R_5 = \boxed{46 - j2\ \Omega}, \qquad R_3 = R_4 = \boxed{53 + j10\ \Omega}$$

---

*<u>11-14</u> Impedance and gain of 2-element array.

(a) This is a single-section W8JK array

From Sec. 10-5,

$$Z_{12} = \boxed{52 - j21\ \Omega}$$

(b) From (11-5-8) and assuming losses,

$$G_f(\phi)(\max)\ (A/HW) = [(2 \times 73)/(73 - 52)]^{\frac{1}{2}} \sin 36^\circ$$
$$= 2.64 \times .588 = \boxed{1.55} \text{ or } 3.8 \text{ dB}$$
$$= 6.0 \text{ dBi}$$

<u>11-16</u> Square array. $\lambda/4$ on a side.

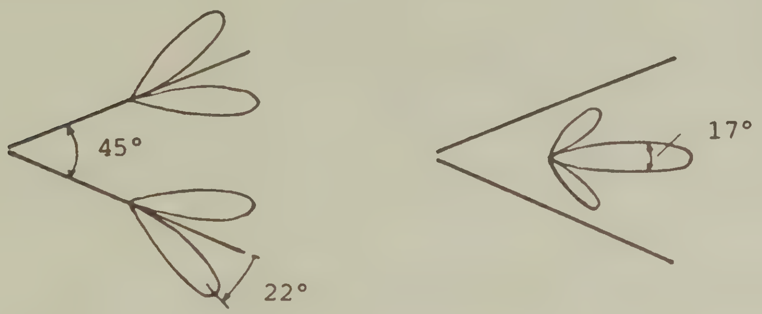
Pattern is a rounded square.

$$E_n = 1.00 \text{ at } \phi = 0^\circ, \pm 90^\circ, 180^\circ$$

$$E_n = 0.895 \text{ at } \phi = \pm 45^\circ, \pm 135^\circ$$

***11-17** **Terminated V. Traveling wave.**

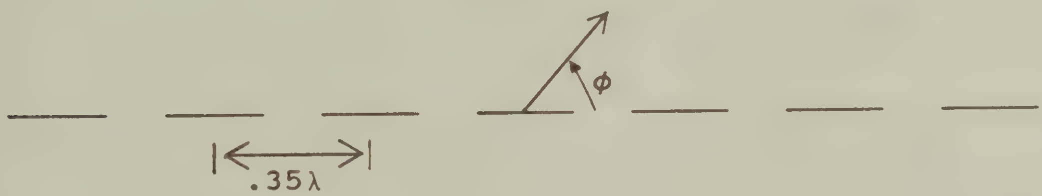
(a) The field pattern for each leg of the V is shown at the left and the combined field pattern at the right. Minor lobes are neglected except for the principal side lobe of the V.



(b) HPBW ≅ $\boxed{17^\circ}$

***11-18** **Seven short dipoles. 4-dB angle.**

The dipoles are assumed to be aligned colinearly so that the pattern of a single dipole is proportional to sin ϕ where ϕ is the angle from the array. Thus,



Since the dipoles are in-phase, the maximum field is at ϕ = 90° or $\phi(E_{max}) = 90^\circ$.

The normalized pattern is given by

$$E_n = \frac{1}{n} \frac{\sin n\psi/2}{\sin \psi/2} \sin \phi \qquad (1)$$

continued

*11-18 continued

where n = 7

$$\psi = (2\pi/\lambda) \times 0.35\lambda \cos\phi$$

$$4 = 20 \log x; \quad x = 1.585$$

Therefore, $E_n(-4dB) = 1/1.585 = 0.631$

Setting (1) equal to 0.631, n = 7 and solving (see note below) yields

$$\phi(-4dB) = 78.3^o$$

Angle from ϕ(Emax) is

$$90^o - 78.3^o = \boxed{11.7^o}$$

Note: Use trial and error to solve (1) for ϕ(-4dB) or calculate pattern with small increments in ϕ. The Appendix B-2 program may be used, modified with dipole pattern factor included by changing line 80 to read: R=MF*ABS(R)*SIN(A) and requesting PRINT instead of PLOT in line 90 to obtain data of required accuracy.

11-19 Square array. $\lambda/2$ on a side.

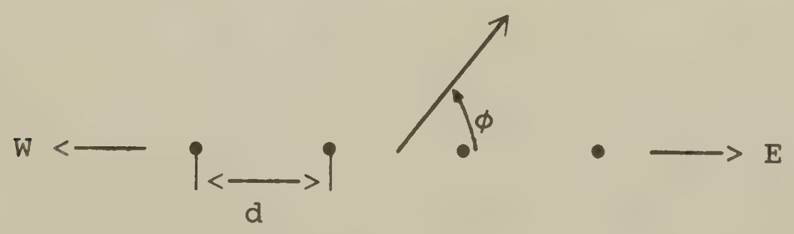
Pattern maxima at $\phi = \pm 45^o, \pm 135^o$

Pattern minima at $\phi = 0^o, \pm 90^o, 180^o$

---

*11-22 Four-tower broadcast array.

(a)



*11-22 continued

Null at $\phi = 90^\circ$ requires that $\delta = \pm 90^\circ$ or $\pm 180^\circ$

For maximum field (fields of all towers in phase) set

$$\Psi = \beta d \cos \phi_{max} + \delta = 0 \qquad \text{and } \delta = -90^\circ = -\pi/2 \text{ rad}$$

so

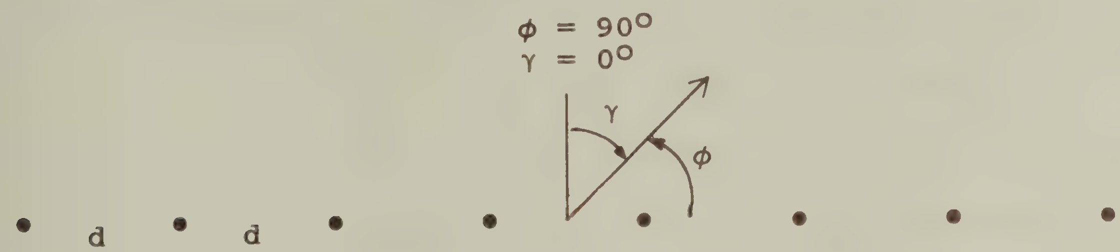
$$d = -\frac{\delta}{\beta \cos \phi_{max}} = \frac{\pi/2}{(2\pi/\lambda) \cos 45^\circ} = \boxed{0.354\ \lambda}$$

If $\delta = -180^\circ = -\pi$ rad,

$$d = \frac{\pi}{(2\pi/\lambda) \cos 45^\circ} = 0.707\lambda \text{ but this exceeds } 0.5\lambda$$

(b) Therefore, $\delta = \boxed{-90^\circ}$

11-23 Eight-source scanning array.



Broadside is at $\phi = 90^\circ$.

Set $\Psi = \beta d \cos \phi_{max} + \delta = 0$

(a) Therefore,

$$\delta = -\frac{2\pi}{\lambda} d \cos \phi_{max} = -\frac{2\pi}{\lambda}\frac{\lambda}{2} \cos 95^\circ = \boxed{+15.7^\circ}$$

$$\delta = -180^\circ \cos 85^\circ = \boxed{-15.7^\circ}$$

Thus, depending on whether δ is $+15.7^\circ$ or -15.7°, the beam is 5° left or right of broadside. continued

11-23 continued

In same way, we have

(b) $\delta = \boxed{\pm 31.3^\circ}$ for beam 10° left or right of broadside

(c) $\delta = \boxed{\pm 46.6^\circ}$ for beam 15° left or right of broadside

(d) From (4-7-7) the angle of the first null from broadside, when the sources are in-phase ($\delta = 0$), is given by the complementary angle

$$\gamma_0 = 90 - \phi_0 = \sin^{-1}(\lambda/nd) = \sin^{-1}(1/4) = 14.48^\circ$$

Therefore, $\quad \text{BWFN} = 2 \times 14.48 = 28.96 \cong \boxed{29^\circ}$

From the long broadside array equation (4-7-10),

$$\text{BWFN} \cong 2\lambda/nd = 1/2 \text{ rad} = 180^\circ/2\pi = 28.65^\circ$$

The HPBW is a bit less than BWFN/2. For long broadside arrays, we have from Table 4-3 (p. 152) that

$$\text{HPBW} = 50.8^\circ/L_\lambda = 50.8/3.5 = 14.5^\circ$$

---

*11-26 E-type rhombic. α given.

From Table 11-1 (p. 507) for a maximum E rhombic,

$$H_\lambda = 1/(4 \sin \alpha) = 1/(4 \sin 17.5^\circ) = \boxed{0.83}$$

$$\phi = 90^\circ - \alpha = \boxed{72.5^\circ}$$

$$L_\lambda = 0.5/\sin^2 \alpha = \boxed{5.5}$$

11-27 **Alignment rhombic.** α given.

From Table 11-1 (p. 507) for an alignment rhombic,

$$H_\lambda = 1/(4 \sin 17.5^\circ) = \boxed{0.83}$$

$$\phi = 90^\circ - \alpha = \boxed{72.5^\circ}$$

$$L_\lambda = 0.371/\sin^2 \alpha = \boxed{4.1}$$

***11-28** **Compromise rhombic.** α and H given.

From Table 11-1 (p. 507) for a compromise rhombic,

$$H_\lambda = = \boxed{0.5}$$

$$\phi = 90^\circ - 17.5^\circ = \boxed{72.5^\circ}$$

$$L_\lambda = \frac{\tan[(\pi L_\lambda)\sin^2 17.5^\circ]}{\sin 17.5^\circ}\left[\frac{1}{2\pi \sin 17.5^\circ} - \frac{0.5}{\tan(\pi \sin 17.5^\circ)}\right]$$

or

$$\frac{L_\lambda}{\tan(16.3^\circ L_\lambda)} = 0.56$$

By trial and error, $L_\lambda = \boxed{5.14}$

11-29 **Compromise rhombic.** α and L given.

From Table 11-1 (p. 507),

$$H_\lambda = 1/(4 \sin 17.5^\circ) = \boxed{0.83}$$

$$\phi = \sin^{-1}\left[\frac{3 - 0.371}{3\cos 17.5^\circ}\right] = \boxed{67^\circ}$$

*__11-30__ Compromise rhombic. α, H and L given.

From Table 11-1 (p. 507, bottom entry),

$$\frac{H_\lambda}{\sin\phi\tan\alpha\tan(2\pi H_\lambda\sin\alpha)} = \frac{1}{4\pi\Psi} - \frac{L_\lambda}{\tan(\Psi 2\pi L_\lambda)}$$

where $\Psi = (1 - \sin\phi\cos\alpha)/2$

By trial and error, $\phi \cong \boxed{60^\circ}$

---

*__11-34__ Sixteen source broadside array.

(a) From (4-6-9),

$$E(HP) = 0.707 = \frac{1}{16}\,\frac{\sin(1440^\circ\cos\phi)}{\sin(90^\circ\cos\phi)}$$

By trial and error,

$\phi = 86.82^\circ$

and HPBW $= 2(90^\circ - 86.82^\circ) = \boxed{6.36^\circ = 6^\circ\ 22'}$

(b) From (4-9-10),

K = 1 (first minor lobe)

$$E_{ML} \cong \frac{1}{16\sin[(2+1)\pi/32]} = 0.215 \text{ or } -13.3\text{ dB}$$

This is only approximate (becomes exact only for very large n).

To determine the level more accurately, we find the approximate angle for the maximum of the first minor lobe from (4-9-5).

$$\phi_m \cong \cos^{-1}\frac{\pm(2K+1)}{2n d_\lambda} = \cos^{-1}\frac{\pm 3}{2 \times 16 \times \frac{1}{2}} = 79.2^\circ$$

Then from (4-6-9) we calculate E at angles close to 79.2° and find that E peaks at 79.7° with

$$E = 0.22012 \quad \text{or} \quad \boxed{-13.15\text{ dB}}$$

Although (4-9-5) locates the angle where the numerator of (4-9-5) is a maximum (=1), the denominator is not constant. See discussion of Sec. 4-9 (p. 156) and also Fig. 4-28 (p. 157).

(c) From the equation for D in Prob. 4-32, the summation term is zero for $d = \lambda/2$ so that $D = 16$ exactly.

Since $D = 4\pi/\Omega_A$, $\Omega_A = 4\pi/D = 4\pi/16 = \boxed{\pi/4 \text{ sr}}$

(d)

HPBW $\cong 1/nd_\lambda = 1/(16 \times 0.5) = 1/8$ rad in ϕ direction

BW in θ direction $= 2\pi$ rad

Therefore,

$$\Omega_M = 2\pi \times (1/8) = \pi/4 \text{ sr and } \epsilon_M = \Omega_M/\Omega_A = 1 \text{ or } 100\%$$

This result is too large since with any minor lobes ϵ_M must be less than unity (or $\Omega_M < \Omega_A$).

For an exact evaluation, we have from Prob. 4-32 that

$$\Omega_M = \frac{2}{n^2 d_\lambda} \left[n\pi d_\lambda \cos\theta + \sum_{k=1}^{n-1} \frac{n-k}{k} \sin(2\pi k d_\lambda \cos\theta) \right]_{\theta_1}^{\theta_2}$$

where $\theta_1 = 90^\circ - \gamma_o$

$\theta_2 = 90^\circ + \gamma_o$

γ_o = angle to first null

From (4-7-7),

$$\gamma_o = \sin^{-1}(1/nd_\lambda) = \sin^{-1}[1/(16 \times 0.5)]$$

$$= \sin^{-1}(1/8) = 7.18^\circ$$

Therefore, $\theta_1 = 82.82^\circ$

$\theta_2 = 97.18^\circ$

continued

*<u>11-34</u> continued

Thus,

$$\Omega_M = \frac{4}{n^2 d_\lambda} \left[n\pi d_\lambda \cos\theta + \sum_{k=1}^{n-1} \frac{n-k}{k} \sin(2\pi k d_\lambda \cos\theta) \right]_{90^\circ}^{97.18^\circ}$$

$$|\Omega_M| = \frac{4}{n^2 d_\lambda} \left[0.125 n\pi d_\lambda + \sum_{k=1}^{n-1} \frac{n-k}{k} \sin(0.25\pi k d_\lambda) \right]$$

$$= \frac{0.5\pi}{16} + \frac{4}{16^2 \times 0.5} \left[\frac{15}{1} \sin(0.125\pi) + \frac{14}{2} \sin(0.25\pi) \right.$$

$$+ \frac{13}{3} \sin(0.375\pi) + \frac{12}{4} \sin(0.5\pi) + \frac{11}{5} \sin(0.625\pi)$$

$$+ \frac{10}{6} \sin(0.75\pi) + \frac{9}{7} \sin(0.875\pi) + \frac{8}{8} \sin(1.00\pi)$$

$$+ \frac{7}{9} \sin(1.25\pi) + \frac{6}{10} \sin(1.375\pi) + \frac{5}{11} \sin(1.5\pi)$$

$$+ \frac{4}{12} \sin(1.625\pi) + \frac{3}{13} \sin(1.75\pi) + \frac{2}{14} \sin(1.875\pi)$$

$$\left. + \frac{1}{15} \sin(2.00\pi) \right]$$

$$= 0.0982 + 0.03125\,[5.740 + 4.950 + 4.003 + 3.000 + 2.033$$

$$+ 1.179 + 0.492 + 0 - 0.550 - 0.554 - 0.455 - 0.308$$

$$- 0.163 - 0.055 + 0]$$

$$= 0.0982 + 0.03125 \times 19.312 = 0.702 \text{ sr} = \Omega_M$$

$$\epsilon_M = \Omega_M/\Omega_A = 0.702/(\pi/4) = \boxed{0.894}$$

***11-34** continued

By graphical integration (see, for example, Sec. 3-14 and Fig. 3-16b) ϵ_M was found to be approximately 0.90, in good agreement with the above result. The graphical integration took a fraction of the time of the above analytical integration and although less accurate, provided confidence in the result because it is much less susceptible to gross errors.

(e) As noted in (c), $D = \boxed{16}$

(f) From $D = 4\pi A_{em}/\lambda^2$,

$$A_{em} = D\lambda^2/4\pi = 16\lambda^2/4\pi = \boxed{1.27\lambda^2}$$

***11-40** **Pattern smoothing.**

$$\text{Ratio} = \boxed{1/2}$$

11-43 **Number of elements.**

From Fig. 11-78,

$$2/(nd_\lambda) = (1/4) \times (1/d_\lambda)$$

or

$$n = \boxed{8}$$

CHAPTER 12

REFLECTOR ANTENNAS AND THEIR FEED SYSTEMS

12-1 Flat sheet reflector.

From (12-2-1) the gain over a $\lambda/2$ reference dipole is given by

$$G_f(\phi) = 2 \left[\frac{R_{11}}{R_{11} + R_L - R_{12}}\right]^{1/2} |\sin (S_r \cos \phi)| \qquad (1)$$

where S = spacing of dipole from reflector
ϕ = angle from perpendicular to reflector

(See Figure 12-2.)

Note that (1) differs from (12-2-1) in that $R_L = 0$ in the numerator under the square root sign since the problem requests the gain to be expressed with respect to a <u>lossless</u> reference antenna.

Maximum radiation is at $\phi = 0$, so (1) becomes

$$G_f(\phi) = 2 \left[\frac{73.1}{73.1 + R_L - 29.4}\right]^{1/2} \sin (2\pi \times 0.15)$$

and for $R_L = 0$

$$G_f(\phi) = \boxed{2.09} \text{ or } 6.41 \text{ dB } (= 8.56 \text{ dBi})$$

Note that R_{12} is for a spacing of 0.3λ ($= 2 \times .15\lambda$)

See Table 10-1.

For $R_L = 10\ \Omega$, $G_f(\phi) = \boxed{1.89}$ or 5.52 dB (= 7.67 dBi)

Note that $G_f(\phi)$ is the gain with respect to a reference $\lambda/2$ dipole and more explicitly can be written $G_f(\phi)$ [A/HW].

The loss resistance $R_L = 10\ \Omega$ results in about 0.9 dB reduction in gain with respect to a lossless reference dipole. If the reference dipole also has $10\ \Omega$ loss resistance, the gain reduction is about 0.3 dB.

12-1 continued

The above gains agree with those shown for $R_L = 0$ and extrapolated for $R_L = 10\ \Omega$ at $S = 0.15\lambda$ in Fig. 12-4. Note that in Fig. 12-4 an equal loss resistance is assumed in the reference antenna.

The pattern for $R_L = 0$ should be intermediate to those in Fig. 12-3 for spacings of 0.125λ $(=\lambda/8)$ and 0.25λ $(=\lambda/4)$. The pattern for $R_L = 10\ \Omega$ is smaller than the one for $R_L = 0$ but of the same shape (radius vector differing by a constant factor).

12-2 Square-corner reflector. Mutual resistances.

From (12-3-6) the gain of a lossless corner reflector over a reference $\lambda/2$ dipole is given by

$$G_f(\phi) = 2\left[\frac{R_{11}}{R_{11} + R_{14} - 2R_{12}}\right]^{1/2} \left|[\cos(S_r \cos\phi) - \cos(S_r \sin\phi)]\right|$$

For $S = \lambda/2$ and maximum radiation direction ($\phi = 0^\circ$) this becomes

$$G_f(\phi) = 4\left[\frac{73.1}{73.1 + 3.8 + 2 \times 24}\right]^{1/2} = 3.06 \text{ or } 9.7 \text{ dB } (= 11.9 \text{ dBi})$$

See Table 10-1 and Fig. 10-12 for the mutual resistance values for R_{14} at 1λ separation and R_{12} at 0.707λ separation. The above calculated gain agrees with the value shown by the curve in Fig. 12-11. The pattern should be identical to the one in Fig. 12-12a.

---

*12-6 Square-corner reflector.

(a) From (12-3-6) the normalized field pattern for $S = 0.35\lambda$ is

$$E_n(\phi) = \left|\frac{[\cos(126^\circ \cos\phi) - \cos(126^\circ \sin\phi)]}{1.588}\right|$$

(b) $R_r = R_{11} + R_{14} - 2R_{12} = 73.1 - 24.8 + 25 = \boxed{73.3\ \Omega}$

continued

*12-6 continued

(c) From (12-3-6) for $\phi = 0$ and $S = 0.35\lambda$

$$G_f(\phi) = 2\ (73.1/73.3)^{\frac{1}{2}} \times 1.588 = 3.17 \text{ or } \boxed{10.0 \text{ dB}}$$

(= 12.1 dBi)

12-7 Square-corner reflector versus array of its image elements.

(a) 4-lobed pattern as in Fig. 12-9 with shape of pattern of Prob. 12-6a.

(b) $R_r = \boxed{73.3\ \Omega}$

(c) $G_f(\phi) = 1.59$ or $\boxed{4.0 \text{ dB}}$ (=6.1 dBi)

since power is fed to all 4 elements instead of to only one. (Power gain down by as factor of 4 or by 6 dB).

---

*12-8 Square-corner reflector array.

(a) From (12-3-6) the gain of one corner reflector with $S = 0.4\lambda$ is given by

$$G_f(\phi) = 2 \left[\frac{73.1}{73.1 - 18.6 + 42}\right]^{1/2} \left|[\cos 144^\circ - \cos 0^\circ]\right|$$

$$= 2 \times 0.870 \times 1.81 = 3.15 \text{ or } \cong 10 \text{ dB } (=12.1 \text{ dBi})$$

Under lossless conditions,

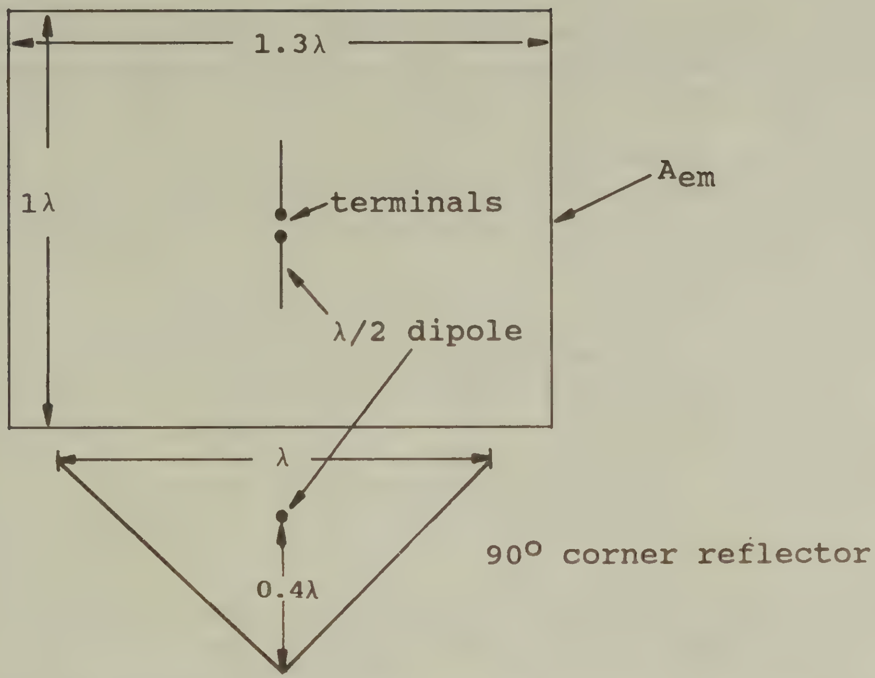
$$D = [G_f(\phi)]^2 \times 1.64 = 16.3$$

Thus, the maximum effective aperture of one corner is

$$A_{em} = \frac{D\lambda^2}{4\pi} \cong \frac{16.3\lambda^2}{4\pi} = 1.3\lambda^2$$

The effective aperture of a single corner may then be represented by a rectangle $1\lambda \times 1.3\lambda$ as in the sketch below.

*12-8 continued



In an array of 4 refectors as in Fig. P12-8 the edges of the apertures overlap 0.3λ so that the reflectors are too close. However, at the 1λ spacing the total aperture is 4λ x 1λ = $4\lambda^2$ and the total gain of the array under lossless conditions is

$$G = D = \frac{4\pi A_{em}}{\lambda^2} = 4\pi \times 4 \cong 50 \text{ or } \boxed{17 \text{ dBi}}$$

No interaction between corner reflectors has been assumed. With wider spacing (=1.3λ) the expected gain = 16.3 x 4 ≅ 65 or 18 dBi.

(b) Assuming a uniform aperture distribution, the HPBW is given approximately from Table 4-3 by

$$\text{HPBW} \cong 50.8^\circ/L_\lambda = 50.8^\circ/4 = 12.7^\circ$$

To determine the HPBW more accurately, let us use the total antenna pattern. By pattern multiplication it is equal to the product of an array of 4 in-phase isotropic point sources with 1λ spacing and the pattern of a single corner reflector as given by

$$E_n(\phi) = \frac{1}{4}\,\frac{\sin(4\pi\sin\phi)}{\sin(\pi\sin\phi)}\,\frac{1}{1.809}\,\left|[\cos(0.8\pi\cos\phi) - \cos(0.8\pi\sin\phi)]\right|$$

continued

***12-8** continued

The 1/4 is the normalizing factor for the array and 1/1.809 for the corner reflector. Thus, when $\phi = 0^\circ$, $E_n(\phi) = 1$. Note that ϕ must approach zero in the limit in the array factor to avoid an indeterminate result.

Half of the above approximate HPBW is $12.7^\circ/2 = 6.35^\circ$. Introducing it into the above equation yields $E_n(\phi) = 0.703$. For $\phi = 6.30^\circ$, $E_n(\phi) = 0.707$ as tabulated below.

ϕ	$E_n(\phi)$
6.35°	0.703
6.30°	0.707

Thus,

$$\text{HPBW} = 2 \times 6.30 = \boxed{12.6^\circ}$$

The 4-source array factor is much sharper than the corner reflector pattern and largely determines the HPBW.

Returning to part (a) for the directivity, let us calculate its value with the approximate relation of (2-9-4) using the HPBW of part (b) for the H-plane and the HPBW of 78° for the E-plane from Sec. 5-5a, p. 221.

Thus,

$$D \cong \frac{41000}{12.6 \times 78} = 42 \quad (=16\ \text{dBi})$$

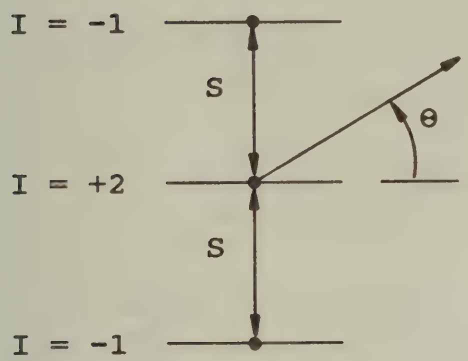
as compared to $D \cong 50$ (=17 dBi) as calculated in part (a).

Although the directivity of 16.3 for a single corner reflector should be accurate, since it is determined from the pattern via the impedances*, the directivity of 50 for the array of 4 corner reflectors involves some uncertainty (apertures overlapping). Nevertheless, the two methods agree within 1 dB.

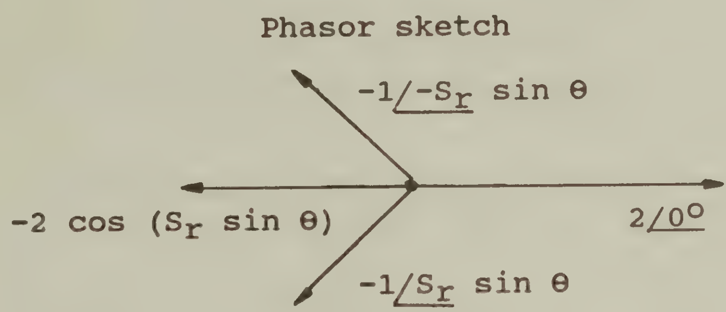
* Assuming infinite sides.

<u>12-10</u> **Square-corner reflector.**

(a) The pattern in the plane of the dipole (E plane) is that of an array of three $\lambda/2$ elements arranged as in the sketch with amplitudes 1:2:1 and phasing as indicated.



Phasor sketch



By pattern multiplication the pattern is the product of the pattern of an array of 3 isotropic sources with amplitudes and phasing -1:+2:-1 and the pattern of $\lambda/2$ dipole (5-5-12). Thus,

$$E = \left[2 - 1\angle S_r \sin\theta \quad -1\angle -S_r \sin\theta\right] \frac{\cos(90^\circ \cos\theta)}{\sin\theta}$$

or, see phasor sketch,

$$E = 2\,[1 - \cos(S_r \sin\theta)]\ \frac{\cos(90^\circ \cos\theta)}{\sin\theta}$$

Dropping the scale factor 2 yields the result sought, q.e.d.

<u>12-11</u> **Corner reflector. $\lambda/4$ to the driven element.**

For the case of no losses,

$D = [G_f(\phi)]^2 \times 1.64$, and for $S = \lambda/4$ and $\phi = 0$,

(12-3-6) becomes $\quad G_f(\phi) = 2\left[\dfrac{73.1}{73.1 - 12.7 - 35}\right]^{1/2} = 3.39$

Therefore, $D = [G_f(\phi)]^2 \times 1.64 = 18.9$ or $\boxed{12.8\ \text{dBi}}$

12-12 Corner reflector. $\lambda/2$ to the driven element.

(a) From Prob. 12-10 the pattern in the E-plane is given by

$$E_n(\theta) = \frac{1}{2}\,[1 - \cos(\pi \sin\theta)]\,\frac{\cos(90^\circ \cos\theta)}{\sin\theta} \qquad (1)$$

From (12-3-6) the pattern in the H-plane is given by

$$E_n(\phi) = \frac{1}{2}\left|\,[\cos(\pi\cos\phi) - \cos(\pi\sin\phi)]\,\right| \qquad (2)$$

Note that $E_n(\theta)$ = maximum for $\theta = 90^\circ$ while $E_n(\phi)$ = maximum for $\phi = 0^\circ$.

(b) Assuming initially that HPBW(θ) $\cong$ HPBW(ϕ) and noting from Fig. 12-11 that for $S = \lambda/2$ the directivity is about 12 dBi, we have from (2-9-4) that

$$D \cong \frac{41000}{\text{HPBW}(\theta)^2} \cong 16 \text{ or HPBW}(\theta) \cong 51^\circ$$

and $$\frac{\text{HPBW}(\theta)}{2} = \frac{51^\circ}{2} \cong 25^\circ$$

Introducing $\theta = 90^\circ - 25^\circ = 65^\circ$ in (1) yields $E_n(\theta)$ which is too high. By trial and error, we obtain $E_n(\theta) \cong 0.707$ when $\theta = 34.5^\circ$.

Therefore, $\text{HPBW}(\theta) \cong 2 \times 34.5^\circ = \boxed{69^\circ}$

Introducing $\phi = 25^\circ$ in (2) yields $E_n(\phi) = 0.60$ which is too low. By trial and error, we obtain $E_n(\phi) \cong 0.707$ when $\phi = 21^\circ$.

Therefore, $\text{HPBW}(\phi) \cong 2 \times 21^\circ = \boxed{42^\circ}$

(c) The terminal impedance of the driven element is (see Prob.12-2 solution),

$$R_T = 73.1 + 3.8 + 2 \times 24 = \boxed{125\ \Omega}$$

12-12 continued

(d) From Prob. 12-2 solution,

$$D = [G_f(\phi)]^2 \times 1.64 = \boxed{15.4} \quad \text{(11.9 dBi) by impedances}$$

$$D \cong \frac{41000}{69^\circ \times 42^\circ} = \boxed{14.1} \quad \text{(=11.5 dBi) by beam widths}$$

The D= 15.4 value is, of course, more accurate since it is based on the pattern via the impedances. The two methods differ, however, by only 0.4 dB.

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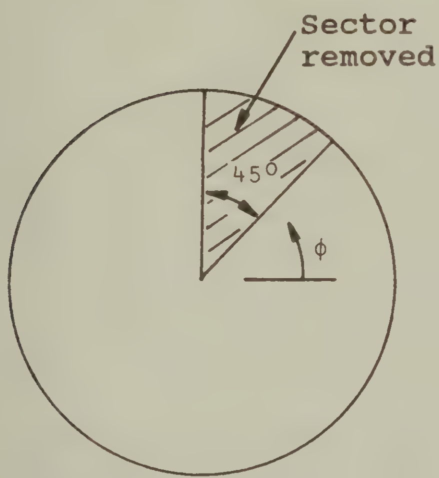
*12-13 Parabolic reflector with missing sector. Effective Aperture.



The full dish has an effective aperture A_e= 100 m^2. Assuming that the dish characteristics are independent of angle (ϕ), removing one 45° sector reduces the effective aperture to 7/8 of its original value provided the feed is modified so as not to illuminate the area of the missing sector. However, the feed is not modified and, therefore, its efficiency is down to 7/8. Therefore, the net aperture efficiency is $(7/8)^2$ and the net effective aperture is

$$(7/8)^2 \times 100 = \boxed{76.6\ m^2}$$

---

*12-14 Efficiency of rectangular aperture with partial taper. Aperture efficiency and directivity.

From Problem 12-17 solution

(a) $\epsilon_{ap} = \boxed{0.81}$ or 81%

(b) $D = 4\pi \times 10 \times 20 \times 0.81 = \boxed{2036}$ or 33 dBi

*<u>12-15</u> **Efficiency of rectangular aperture with full taper. Aperture efficiency and directivity.**

From Problem 12-18 solution

(a) $\epsilon_{ap} = \boxed{0.657} \cong 66\%$

(b) $D = 4\pi \times 10 \times 20 \times 0.657 = \boxed{1651}$ or 32 dBi

---

<u>12-16</u> **Efficiency of aperture with phase ripple.**

Referring to Sec. 12-9,

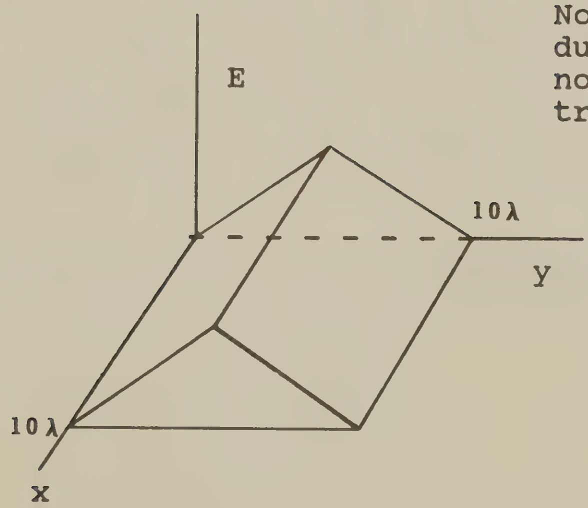
let

$$\underbrace{E'(x, y)_{max}}_{\text{Design field}} = \underbrace{E(x, y)_{max}}_{\text{Actual field}} = 1$$

Design:

$$E'_{av} = \frac{1}{A_p} \int\int E'(x,y)\, dxdy \tag{1}$$

$$= \frac{2}{A_p} \int_0^{10\lambda} \int_0^{5\lambda} \frac{y}{5\lambda}\, dxdy = 1/2$$



Note: This result can be deduced directly from figure by noting that average height of triangle is 1/2 max.

(b) Utilization factor, k_u:

$$k_u = \frac{1}{\frac{1}{A_p}\int\int \left[\frac{E'(x,y)}{E'_{av}}\right]\left[\frac{E'(x,y)}{E'_{av}}\right]^* dxdy} \qquad (2)$$

$$k_u = \frac{1}{\frac{1}{A_p}\frac{2}{(1/2)^2}\int_0^{10\lambda}\int_0^{5\lambda}\left[\frac{y}{5\lambda}\right]^2 dxdy} = \boxed{3/4} \qquad (3)$$

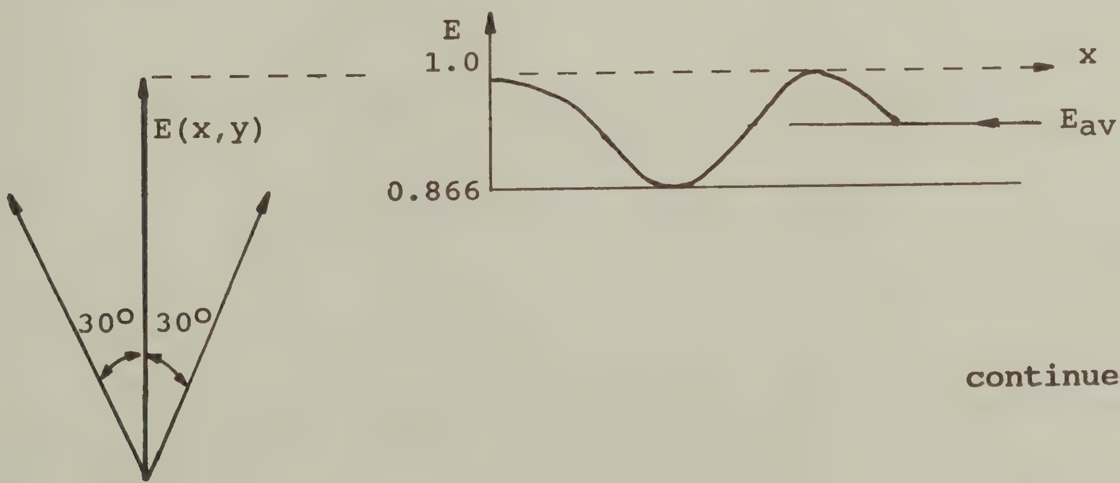
Note that for in-phase fields (12-9-50) is a simplified form of (2) giving

$$\frac{E_{av}^2}{\left[E^2\right]_{av}} = \frac{(1/2)^2}{1/3} = 3/4 = k_u \text{ as in (3).} \qquad (4)$$

(a) Design directivity, D(design):

$$D(\text{design}) = (4\pi/\lambda^2)(A_p k_u) = (4\pi/\lambda^2)(100\lambda^2)(3/4) = \boxed{940} \qquad (5)$$

Turning attention now to the effect of the phase variation:



continued

<u>12-16</u> continued

$$E_{av} = \frac{2}{A_p} \int_0^{10\lambda} \cos\left[\frac{\pi}{6} \sin\frac{2\pi x}{\lambda}\right] dx \int_0^{5\lambda} \frac{y}{5\lambda}\, dy \qquad (6)$$

$$= (1/2)\ 0.933$$

Note that from figures above,

$$2E_{av} \cong \frac{1 + 0.866}{2} = 0.933$$

(d) Achievement factor, k_a:

$$k_a = \frac{\dfrac{1}{A_p} \displaystyle\int\!\!\int \left[\frac{E'(x,y)}{E'_{av}}\right] \left[\frac{E'(x,y)}{E'_{av}}\right]^* dxdy}{\dfrac{1}{A_p} \displaystyle\int\!\!\int \left[\frac{E(x,y)}{E_{av}}\right] \left[\frac{E(x,y)}{E_{av}}\right]^* dxdy}$$

$$k_a = \frac{(4/3)}{\dfrac{1}{A_p}\, \dfrac{1}{[(1/2)\ 0.933]^2}\ 2 \displaystyle\int\!\!\int (y/5\lambda)^2\, dxdy} = \boxed{0.87} \qquad (7)$$

where $E(x)\ E^*(x) = 1$

Note that gain loss due to total phase variation across aperture (<u>not</u> surface deviation) is from (12-10-3)

$$k_g = \cos^2(360^\circ \frac{\delta'}{\lambda})$$

where $\dfrac{\delta'}{\lambda} = \dfrac{30^\circ \times .707}{360^\circ} = \dfrac{21.2^\circ}{360^\circ}$

or $k_g = \cos^2 21.2^\circ = 0.87 = k_a$ as in (7)

(c) Directivity:

$$D = (4\pi A_p/\lambda^2)\ k_u\ k_a = 4\pi \times 100 \times (3/4) \times 0.87 = \boxed{818}$$

continued

12-16 continued

(e) Effective aperture, A_e:

$$A_e = (\lambda^2/4\pi)\, D = A_p k_u k_a = \boxed{65.2\ \lambda^2}$$

(f) Aperture efficiency, ϵ_{ap}:

$$\epsilon_{ap} = k_a\, k_u = \boxed{0.65}$$

NOTE: Although phase errors with small correlation distance ($\cong\lambda$) as is Prob. 12-16 reduce the directivity and, hence, increase Ω_A, the HPBW is not affected appreciable. However, for larger correlation distances ($>>\lambda$) the scattered radiation becomes more directive, causing the near side lobes to increase and ultimately the main beam and the HPBW may be affected.

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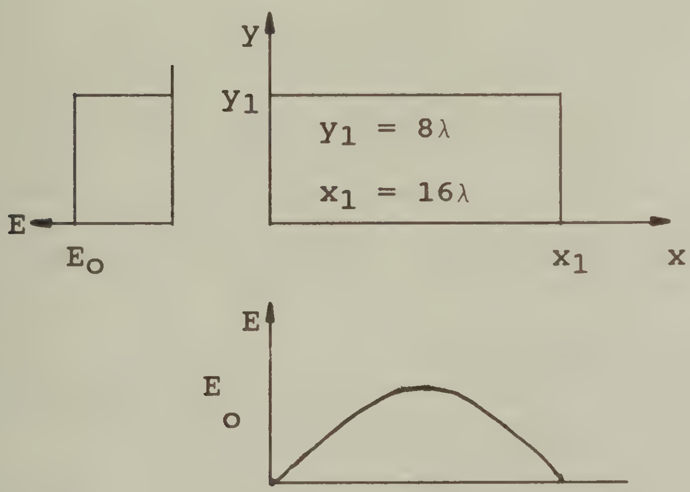
*12-17 Rectangular aperture. Cosine taper. Aperture efficiency and directivity.



$$E(x) = E_o \sin \frac{\pi x}{x_1}$$

Although the taper in the x-direction is described as a cosine taper, let us represent it by a sine function as follows:

(a) From (12-9-50),

$$\epsilon_{ap} = \frac{[E(x)_{av}]^2}{[E^2(x)]_{av}}$$

continued

*12-17 continued

where

$$E(x)_{av} = \frac{1}{x_1}\int_0^{x_1} E(x)\ dx = \frac{E_o}{x_1}\int_0^{x_1} \sin\frac{\pi x}{x_1}\ dx$$

$$= (E_o/x_1)(-x_1/\pi)\ \cos\ (\pi x/x_1)\ \Big|_0^{x_1} = (2E_o/\pi)$$

$$[E^2(x)]_{av} = (1/x_1)\int_0^{x_1} E^2(x)\ dx = (E_o^2/x_1)\int_0^{x_1} \sin^2(\pi x/x_1)\ dx$$

$$= (E_o^2/2)$$

Therefore, $$\epsilon_{ap} = \frac{\left[\frac{2}{\pi}\ E_o\right]^2}{\frac{1}{2}\ E_o^2} = \frac{8}{\pi^2} = \boxed{0.811} \text{ or } \cong 81\%$$

(b) $A_e = \epsilon_{ap}\ A_{em} = 0.81 \times 8\lambda \times 16\lambda = 103.7\lambda^2$

$$D = (4\pi A_e/\lambda^2) = (4\pi \times 103.7\lambda^2)/\lambda^2 = \boxed{1304} \text{ or } 31.2 \text{ dBi}$$

12-18 Rectangular aperture. Cosine tapers. Aperture efficiency and directivity.

Let the distribution be represented by

$$E(x,y) = E_o\ \sin\frac{\pi x}{x_1}\ \sin\frac{\pi x}{y_1}$$

(a) $$E(x,y)_{av} = \frac{1}{x_1 y_1}\int_0^{x_1}\int_0^{y_1} E(x,y)\ dxdy$$

$$= \frac{E_o}{x_1 y_1}\int_0^{x_1} \sin\frac{\pi x}{x_1}\ dx \int_0^{y_1} \sin\frac{\pi y}{y_1}\ dy$$

12-18 continued

$$E(x,y)_{av} = \frac{E_o}{x_1 y_1}\left[-\frac{x_1}{\pi}\cos\frac{\pi x}{x_1}\right]_0^{x_1}\left[-\frac{y_1}{\pi}\cos\frac{\pi y}{y_1}\right]_0^{y_1}$$

$$= 4E_o/\pi^2$$

$$[E^2(x,y)]_{av} = \frac{1}{x_1 y_1}\int_0^{x_1}\int_0^{y_1} E^2(x,y)\,dxdy$$

$$= \frac{E_o^2}{x_1 y_1}\int_0^{x_1}\sin^2\frac{\pi x}{x_1}\,dx\int_0^{y_1}\sin^2\frac{\pi y}{y_1}\,dy = E_o^2/4$$

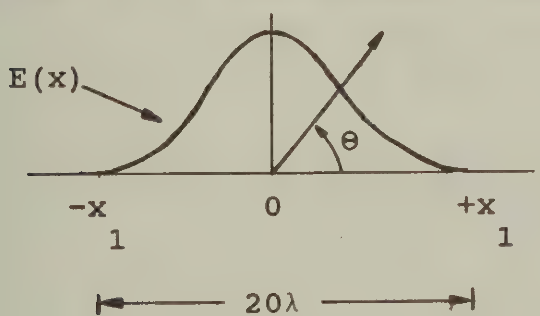
Therefore,

$$\epsilon_{ap} = \frac{\left[\frac{4}{\pi^2}E_o\right]^2}{\frac{1}{4}E_o^2} = \frac{16 x 4}{\pi^4} = \boxed{0.657} \quad \text{or} \cong 66\%$$

(b) $A_e = \epsilon_{ap} A_{em} = 0.657 \text{ x } 8\lambda \text{ x } 16\lambda = 84.1\ \lambda^2$

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \text{ x } 84.1\ \lambda^2}{\lambda^2} = \boxed{1057} \quad \text{or } 30.2 \text{ dBi}$$

***12-19** A 20λ line source. Cosine-square taper.



The field along the line may be represented by

$$E(x) = \cos^2\frac{\pi x}{2x_1}$$

continued

*12-19 continued

(a) The field pattern $E(\theta)$ is the Fourier transform of the distribution $E(x)$ along the line.

Thus,

$$E(\theta) = \int_{-x_1}^{+x_1} E(x)\, e^{j(2\pi x/\lambda)\cos\theta}\, dx$$

$$= \int_{-10\lambda}^{+10\lambda} \cos^2[(\pi/2)(x/10\lambda)]\, e^{j(2\pi x/\lambda)\cos\theta}\, dx$$

Let $s = x/\lambda$ from which $dx = \lambda\, ds$

Then

$$E(\theta) = \lambda \int_{-10}^{+10} \cos^2(\pi s/20)\, e^{j2\pi s\cos\theta}\, ds$$

$$= \lambda \int_{-10}^{+10} \frac{1 + \cos(\pi s/10)}{2}\, e^{j2\pi s\cos\theta}\, ds$$

$$= \frac{\lambda}{2} \int_{-10}^{+10} e^{j2\pi s\cos\theta}\, ds$$

$$+ \frac{\lambda}{2} \int_{-10}^{+10} \cos(\pi s/10)\, e^{j2\pi s\cos\theta}\, ds$$

and

$$E_n(\theta) = \frac{\sin(20\pi\cos\theta)}{2\pi\cos\theta} + \frac{1}{2}\left[\frac{\sin(20\cos\theta + 1)\pi}{[2\cos\theta + (1/10)]\pi}\right.$$

$$\left. + \frac{\sin(20\cos\theta - 1)\pi}{[2\cos\theta - (1/10)]\pi}\right]$$

$$= \boxed{\frac{\sin(20\pi\cos\theta)}{2\pi\cos\theta}\left[1 - \frac{4\cos^2\theta}{4\cos^2\theta - 0.01}\right]} \quad (1)$$

***<u>12-19</u> continued**

(b) From graph or by trial and error from (1)

$$\text{HPBW} = 2\ (90^\circ - 87.9^\circ) = \boxed{4.2^\circ}$$

From Table 4-3 for a 20λ <u>uniform</u> aperture

$$\text{HPBW} = 50.8/L_\lambda = 50.8/20 = 2.5^\circ$$

Thus, the cosine-squared aperture distribution has nearly twice the HPBW of the uniform aperture but its side lobes are much lower with first side lobe down 31 dB as compared to only 13 dB down for a 20λ uniform aperture distribution.

CHAPTER 13

SLOT, HORN AND COMPLEMENTARY ANTENNAS

***13-1** **Boxed-slot impedance.** Complementary dipole.

From (13-6-12) the impedance of an unboxed slot is

$$Z_s = \frac{35476}{R_d + jX_d}$$

where R_d is the resistance and X_d is the reactance of the complementary dipole. Thus,

$$Z_s = \frac{35476}{100 + j0} = 354.8\ \Omega$$

Boxing the slot doubles the impedance so

$$Z_s = 2 \times 354.8 = 709.6 \cong \boxed{710\ \Omega}$$

<u>13-2</u> **Open-slot impedance.** Complementary dipole.

From (13-6-12), $$Z_d = \frac{35476}{Z_s} = \frac{35476}{75} = 473\ \Omega$$

From Fig. 9-12 a center-fed cylindrical dipole with length-to-<u>diameter</u> ratio of $\sim$37 has a resistance at 4th resonance of $\sim$473 Ω (or twice that of a cylindrical stub antenna of a length-to-<u>radius</u> ratio of 37). The width of the complementary slot should be twice the dipole diameter, so it should have a length-to-width ratio of $\sim$181. At 4th resonance the dipole is $\sim 2\lambda$ long and the slot should be the same length. The pattern will be midway between those in Fig. 9-13 (right-hand column, bottom two patterns) but with E and H interchanged.

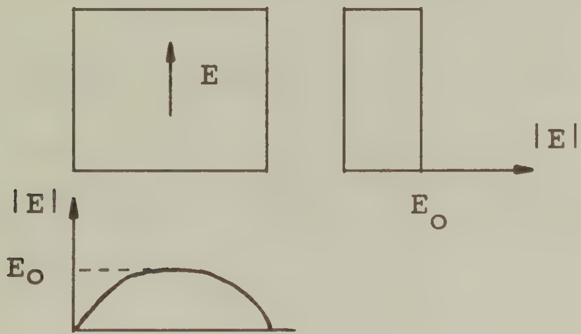
Nothing is mentioned in the problem statement about pattern so the question is left open as to whether this pattern would be satisfactory.

13-2 continued

The above dimensions do not constitute a unique answer, as other shapes meeting the impedance requirement are possible.

---

*13-3 Optimum horn gain.



Assuming a uniform E in the E-direction and a cosine distribution in the H-direction, as in the sketches, and with phase everywhere the same, the aperture efficiency from (12-9-50) is

$$\epsilon_{ap} = \frac{E_{av}^2}{(E^2)_{av}} = (2/\pi)^2 \; E_o^2/(E_o^2/2) = 8/\pi^2 = 0.81$$

A more detailed evaluation of ϵ_{ap} for a similar distribution is given in the solution to Prob. 12-17.

Assuming no losses,

$$\text{Power gain} = D = 4\pi A_e/\lambda^2$$

where $A_e = \epsilon_{ap} \, A_{em} = \epsilon_{ap} \, A_p = 0.81 \times 10^2 \lambda^2 = 81\lambda^2$

and $D = 4\pi \; 81 = 1018$ or 30 dBi

The same gain is obtained by extrapolating the $a_{E\lambda}$ line in Fig. 13-25a to 10λ. However, this makes $a_{H\lambda} > a_{E\lambda}$ and not equal as in this problem.

continued

*13-3 continued

In an optimum horn, the length (which is not specified in this problem) is reduced by relaxing the allowable phase variation at the edge of the mouth by arbitrary amounts ($90^\circ = 2\pi \times .25$ rad in the E-plane and $144^\circ = 2\pi \times .4$ rad in the H-plane). This results in less gain than calculated above, where uniform phase is assumed over the aperture.

From (13-9-2), which assumes 60% aperture efficiency, the directivity of the 10λ square horn is

$$D = 7.5 \times A_p/\lambda^2 = 7.5 \times 10^2 = \boxed{750} \text{ or } 29 \text{ dBi}$$

To summarize: when uniform phase is assumed ($\epsilon_{ap} = 0.81$) as in the initial solution above, D = 1018 or 30 dBi but for an optimum (shorter) horn ($\epsilon_{ap} = 0.6$), D = 750 or 29 dBi.

13-4 Horn pattern.

From (4-14-12) the pattern of a uniform aperture of length a is

$$E_n = \frac{\sin(\psi'/2)}{\psi'/2} = \frac{\sin(\pi a_\lambda \sin\theta)}{\pi a_\lambda \sin\theta} \tag{1}$$

where a = aperture length = 10λ

θ = angle from broadside

(b) From Table 4-3, HPBW = 50.8/10 = 5.08°.

Introducing $5.08/2 = 2.54^\circ$ into (1) yields $E_n = 0.707$ which

confirms that $\boxed{5.08^\circ}$ is the true HPBW since

$$P_n = E_n^2 = 0.707^2 = 0.5$$

Using (4-7-7) and setting $nd_\lambda = a_\lambda$ for a continuous aperture,

$$\text{BWFN} = 2\sin^{-1}(1/a_\lambda) = 2\sin^{-1}(1/10) = \boxed{11.48^\circ}$$

Setting $nd_\lambda = a_\lambda$ assumes n very large and d_λ very small, but we have not assumed that their product nd_λ is necessarily very large. If we had, we could write

$$\text{BWFN} = 2/a_\lambda \text{ rad}$$

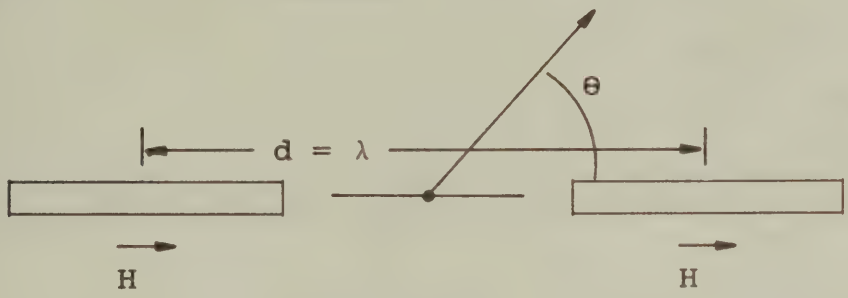
and obtain

$$\text{BWFN} = 2/10 \text{ rad} = 11.46^\circ$$

for a difference of 0.02°.

13-5 Two $\lambda/2$ slots. Broadside array. 1λ spacing between centers.

Thin slots are assumed.



ϕ measured in plane perpendicular to page

The pattern in the E-plane is a circle (E not a function of angle) or $E(\phi) = \boxed{1}$

In the H-plane we have by pattern multiplication that the pattern is the product of 2 in-phase isotropic sources spaced 1λ and the pattern of a $\lambda/2$ slot. The pattern of the $\lambda/2$ slot is the same as for a $\lambda/2$ dipole but with E and H interchanged.

The pattern of the 2 isotropic sources is given by

$$|E| \text{ or } |H| = e^{-j(\beta d/2)\cos\theta} + e^{+j(\beta d/2)\cos\theta}$$

$$= 2\cos[(\beta d/2)\cos\theta] = 2\cos(\pi\cos\theta)$$

continued

The total normalized pattern in the H-plane is then

$$E_n(\theta) = \boxed{\left| \cos(\pi \cos\theta) \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right|}$$

---

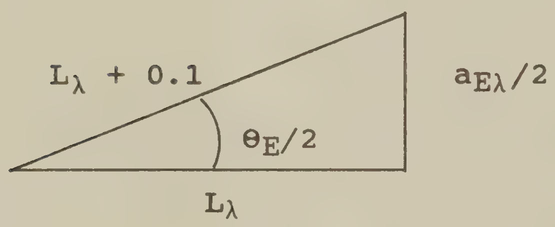
*13-6 Boxed slot. Complementary dipole.

From (13-6-12) we have a boxed slot

$$Z_s = 2 \times \frac{35476}{90 + j10} = \boxed{779 - j87\ \Omega}$$

13-7 Pyramidal horn. Tolerances.

(a) For a 0.1λ tolerance in the E-plane, the relation with dimensions in wavelengths is shown in the sketch.



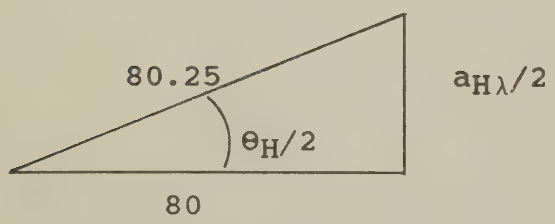
From which,

$$L_\lambda^2 + a_{E\lambda}^2/4 = L_\lambda^2 + .2L_\lambda + .01$$

with $a_{E\lambda} = \underline{8}$ (given),

$$L_\lambda \cong a_{E\lambda}^2/.8 = \boxed{80}$$

In the H-plane we have from the sketch that



$a_{H\lambda}/2 = 6.33$ and $a_{H\lambda} = \underline{12.7}$

$$\theta_E/2 = \tan^{-1} 4/80 = \boxed{2.9^\circ}$$

$$\theta_H/2 = \tan^{-1} 6.33/80 = \boxed{4.5^\circ}$$

13-7 continued

(c) If the phase over the aperture is uniform $\epsilon_{ap} = 0.81$ (see solution to Probs. 12-17 and 13-3),

$$D = 4\pi \times 8 \times 12.7 \times .81 = 1034 \text{ or } 30.1 \text{ dBi}$$

However, the phase has been relaxed to $36^\circ = 2\pi \times 0.1$ rad in the E-plane and to $90^\circ = 2\pi \times 0.25$ rad in the H-plane, resulting in reduced aperture efficiency, so ϵ_{ap} must be less than 0.8. If the E-plane phase is relaxed to 90° and the H-plane phase to 144°, $\epsilon_{ap} \sim 0.6$ which is appropriate for an optimum horn. Thus, for the conditions of this problem which are between an optimum horn and uniform phase, $0.6 < \epsilon_{ap} < 0.8$. Taking $\epsilon_{ap} \cong 0.7$

$$D \cong 4\pi \times 8 \times 12.7 \times .7 = \boxed{894} \text{ or } 29.5 \text{ dBi}$$

(b) Assuming uniform phase in the E-plane,

$$(\text{HPBW})_E \cong 50.8^\circ/a_{E\lambda} = 50.8^\circ/8 = 6.35^\circ \cong \boxed{6.4^\circ}$$

and from the approximation

$$D \cong \frac{41\ 000}{(\text{HPBW})_E(\text{HPBW})_H} = \frac{41\ 000}{6.4\ (\text{HPBW})_H} = 894$$

so $(\text{HPBW})_H \cong 7.2^\circ$

From Table 13-1 for an optimum horn,

$$(\text{HPBW})_E \cong 56^\circ/8 = 7^\circ$$

$$(\text{HPBW})_H \cong 67/12.7 = \boxed{5.3^\circ}$$

The true $(\text{HPBW})_E$ for this problem is probably close to 6.4° while the true $(\text{HPBW})_H$ is probably close to 5.3°.

(d) $\epsilon_{ap} \cong \boxed{0.7}$ from part (c).

CHAPTER 14

LENS ANTENNAS

<u>14-1</u> Dielectric lens. Fermat's principle.

(a) $\lambda = 3 \times 10^8/5 \times 10^9 = 0.06$ m $= 60$ mm

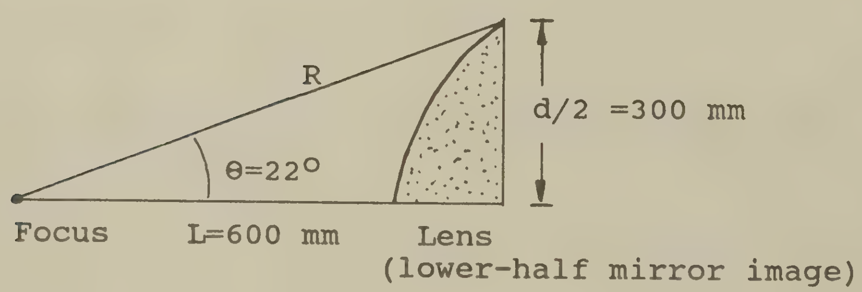
$F = 1$ so $L = d = 10\lambda = 600$ mm (d = diameter)

$n = 1.4$ (see Table 14-1)

Therefore from (14-2-7),

$$R = \frac{(1.4 - 1)\ 600}{1.4 \cos\theta - 1}$$

θ	R	R sin θ
0°	600 mm	0 mm
10°	634	110
20°	761	260
22°	805	$\sim 300 = d/2$



continued

14-1 continued

(b) From (14-2-14), power density at edge of lens is

$$\frac{S_\theta}{S_o} = \frac{(1.4 \cos 22^\circ - 1)^3}{(1.4 - 1)^2 (1.4 - \cos 22^\circ)} = 0.35 \quad \text{or } 4.6 \text{ dB down}$$

To reduce side lobes, this much or even more taper may be desirable. To obtain a uniform aperture distribution, as requested in the problem, requires a feed antenna at the focus with more radiation (up about 4.6 dB) at 22° off axis than on axis. This is difficult to achieve without unacceptable spillover unless the lens is enclosed in a conical horn, except that at edge locations where E is parallel to the edge, E must be zero. To reduce this effect a corrugated horn could be used.

14-2 Artificial dielectric.

From Table 14-2

(a) $\epsilon_r(\text{sphere}) = 1 + 4\pi N a^3 = 1.4$

At 3 GHz, $\lambda = 3 \times 10^8 \text{ m s}^{-1}/3 \times 10^9 \text{ Hz} = 0.1 \text{ m} = 100 \text{ mm}$

For $a \ll \lambda$, take a(radius) = 5 mm from which

$$N = \frac{1.4 - 1}{4\pi a^3} = \frac{0.4}{4\pi(5 \times 10^{-3})^3} = \boxed{255{,}000 \text{ m}^{-3}}$$

The dielectric volume per sphere = $1/255{,}000 = 4 \times 10^{-6} \text{ m}^3$

while the volume of each sphere is given by

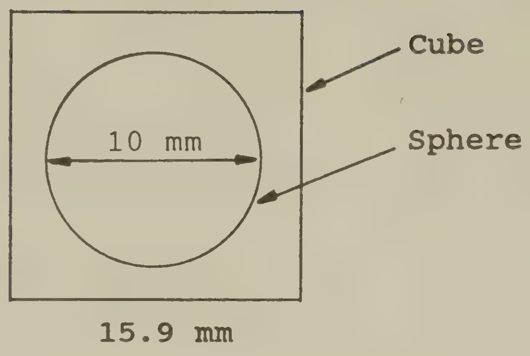
$$\frac{4}{3}\pi a^3 = \frac{4}{3}\pi (5 \times 10^{-3})^3 = 5.2 \times 10^{-7} \text{ m}^3$$

Therefore,

$$\frac{\text{volume of dielectric}}{\text{volume of sphere}} = \frac{4 \times 10^{-6}}{5.2 \times 10^{-7}} = 7.7$$

$$(4 \times 10^{-6})^{1/3} = 1.59 \times 10^{-2} = 15.9 \text{ mm} = \text{side of cube}$$

versus sphere diameter = 2 x 5 = 10 mm



Thus, there is 15.9 - 10 = 5.9 mm between adjacent spheres in a cubical lattice so there is room for the spheres without touching, provided the lattice is uniform.

(b) ϵ_r (discs) = 1 + 5.33 N a^3

Taking a(radius) = 5 mm (diameter = 10 mm),

$$N = \frac{1.4 - 1}{5.33(5 \times 10^{-3})^3} = \boxed{600{,}000 \text{ m}^{-3}}$$

The dielectric volume per disc = $\dfrac{1}{600{,}000}$ = 1.7×10^{-6} m^3

for a cube side length of $(1.7 \times 10^{-6})^{1/3} \cong 12$ mm

so that there is 12 - 10 = 2 mm minimum spacing between adjacent discs in a uniform lattice.

(c) ϵ_r(strips) = 1 + 7.85 N w^2

Taking w (width) = 10 mm,

$$N = \frac{1.4 - 1}{7.85(10^{-2})^3} = \boxed{51{,}000 \text{ m}^{-2}}$$

as viewed in cross section (see Fig. 14-8a). The square area per strip is then

$$1/51{,}000 = 2 \times 10^{-5} \text{ m}^2$$

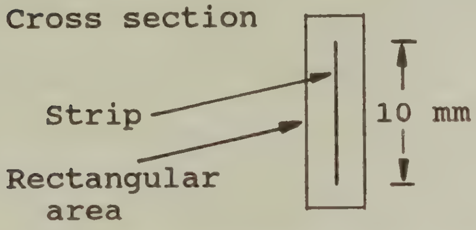
for a cross-sectional area side length

$$(2 \times 10^{-5})^{1/2} = 4.5 \text{ mm}$$

continued

14-2 continued

This is less than the strip width. However, if the square is changed to a rectangle of the same area with side length ratio of 9 as in the sketch, the edges of the strips are separated by 3.5 mm and the flat sides by 1.5 mm.



The above answers are not unique and are not necessarily the best solutions.

***14-3** Unzoned metal-plate lens.

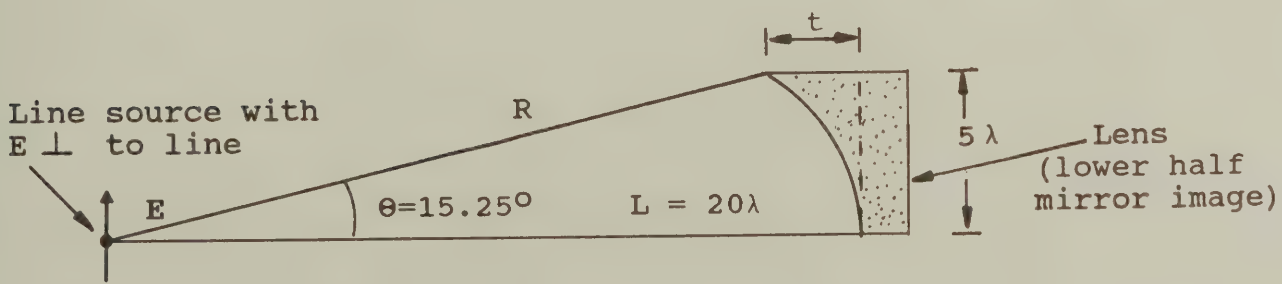
$\lambda = 3 \times 10^8 / 3 \times 10^9 = 0.1$ m = 100 mm

$n = 0.6$, $F = 2$ so $A = L/2$ (Fig. 14-13)

(b) Expressing dimensions in λ, we have from (14-4-4)

$$R_\lambda = \frac{(1-n)\,L_\lambda}{1 - n\cos\theta} = \frac{(1-0.6)\,20}{1-0.6\cos\theta} = \frac{8}{1-0.6\cos\theta}$$

θ	R_λ	$R_\lambda \sin\theta$
0°	20	0
10°	19.6	3.4
15.25°	19.0	5.0



*<u>14-3</u> continued

(a) From (14-4-2),

$$n = [1 - (\lambda_o/2b)^2]^{1/2} \quad \text{or} \quad b = \lambda_o/2(1 - n^2)^{1/2}$$

For $n = 0.6$, $b = 0.625\lambda_o = \boxed{62.5 \text{ mm}}$

(c) From (14-4-12),

Bandwidth $= 2n\delta_\lambda/(1 - n^2)t_\lambda$

$t_\lambda = L_\lambda - R_\lambda \cos\theta = 20 - 19 \cos 15.25^\circ = 1.67$

Therefore,

$$\text{Bandwidth} = \frac{2 \times 0.6 \times 0.25}{(1 - 0.6^2)\ 1.67}$$

$$= 0.28 \quad \text{or} \quad \boxed{28\%}$$

CHAPTER 15

BROADBAND AND FREQUENCY INDEPENDENT ANTENNAS

15-1 Log spiral.

High frequency limit = 10 GHz, $\lambda = 3 \times 10^8/10 \times 10^9 = 30$ mm

Low frequency limit = 1 GHz, $\lambda = 300$ mm

Take $\beta = 77.6^\circ$ (see Fig. 15-4)

From (15-3-5)

$$r = \text{antiln}\,(\theta/\tan\beta) = \text{antiln}\,(\theta/4.55)$$

θ	r	R
0 rad	1	1.5 mm
$\pi/2$	1.41	2.12
π	2.00	3.00
$3\pi/2$	2.82	4.23
2π	4.00	6.00
$5\pi/2$	5.66	8.50
3π	8.00	12.0
$7\pi/2$	11.3	17.0
4π	16.0	24.0
$9\pi/2$	22.6	34.0
5π	32.0	48.0
$11\pi/2$	45.2	68.0
6π	64.0	96.0

continued

15-1 continued

Spiral is like one in Figure 15-4. If gap d at center is equal to $\lambda/10$ at high frequency limit, then gap should be 30/10 = 3 mm and radius R of actual spiral = 3/2 = 1.5 mm.

If diameter of spiral is $\lambda/2$ at low frequency limit, then the actual spiral radius should be 300/(2 x 2) = 75 mm.

This requires that

$$\theta = 4.55 \ln (75/1.5) = 17.8 = 5.7\pi$$

For good measure we make $\theta = 6\pi$. Thus, the spiral has 3 turns ($\theta = 6\pi$).

The table gives data for the actual spiral radius R in mm versus the angle θ in rad. The overall diameter of the spiral is 96 x 2 = 192 mm which at 1 GHz is 192/300 = 0.64λ.

Calling the above spiral number 1, draw an identical spiral rotated through $\pi/2$ rad, a third rotated through π rad and a fourth rotated through $3\pi/2$ rad. Metalize the areas between spirals 1 and 4 and between spirals 2 and 3, leaving the remaining areas open. Connect the feed across the gap at the innermost ends of the spirals as in Fig. 15-5.

15-2 Log periodic.

From Fig. 15-12 let us select the point where $\alpha = 15^\circ$ intersects the optimum design line which should result in an antenna with slightly more than 7 dBi gain. From the figure, k = 1.195. The desired frequency ratio is 5 = F = 250/50. Thus, from (7) the required number of elements must be at least equal to

$$n = \frac{\log F}{\log k} = \frac{\log 5}{\log 1.195} = 9.0$$

at 250 MHz, $\lambda = 1.2$ m, $\lambda/2 = \boxed{0.6 \text{ m}}$

at 50 MHz, $\lambda = 6$ m, $\lambda/2 = \boxed{3 \text{ m}}$

If element 1 is 0.6 in long, then element 10 (= n + 1) is

$0.6 \times 1.195^9 = 2.98$ m or approximately 3 m as required.

15-2 continued

Adding a director in front of element 1 and a reflector in back of element 10 brings the total number of elements to $\boxed{12}$.

The length ℓ_2 of any element with respect to the length ℓ, of the next shorter element is given by

$$\frac{\ell_2}{\ell_1} = k = \boxed{1.195}$$

From (15-5-5) (note geometry of Fig. 15-11), the spacing s between any two elements is related to the length ℓ of the adjacent shorter element by

$$s = \frac{\ell(k-1)}{2 \tan \alpha} = \frac{0.195\ \ell}{2 \tan 15^{\circ}} = \boxed{0.364\ \ell}$$

[Note that (15-5-6) gives s with respect to adjacent longer elements.]

Finally, connect the elements as in Fig. 15-10.

15-3 Stacked LPs.

(a) From the worked example of Sec. 15-5,

$\alpha = 15^{\circ}$, $k = 1.2$, $F = 4$, $n = 7.6\ (8)$ and $n + 1 = 9$

Consider that $\lambda_{min} = 1$ m and $\lambda_{max} = 4$ m.

Therefore, $\ell_1 = 0.5$ m and $\ell_{n+1} = \ell_1 k^n = 0.5 \times 1.2^8 = \ell_9 = 2$ m

From (15-5-6) the distance between elements 1 and 9 is

$$S = \sum_{n=0}^{n-2} \ell_1 \frac{(k-1)\ k^n}{2 \tan \alpha} \qquad \text{where } \ell_1 = 0.5 \text{ m}$$

and

$$S = \sum_{n=0}^{7} \frac{0.2 \times 1.2^n}{4 \tan 15^{\circ}} = 0.187 + 0.224 + 0.268 + 0.322$$

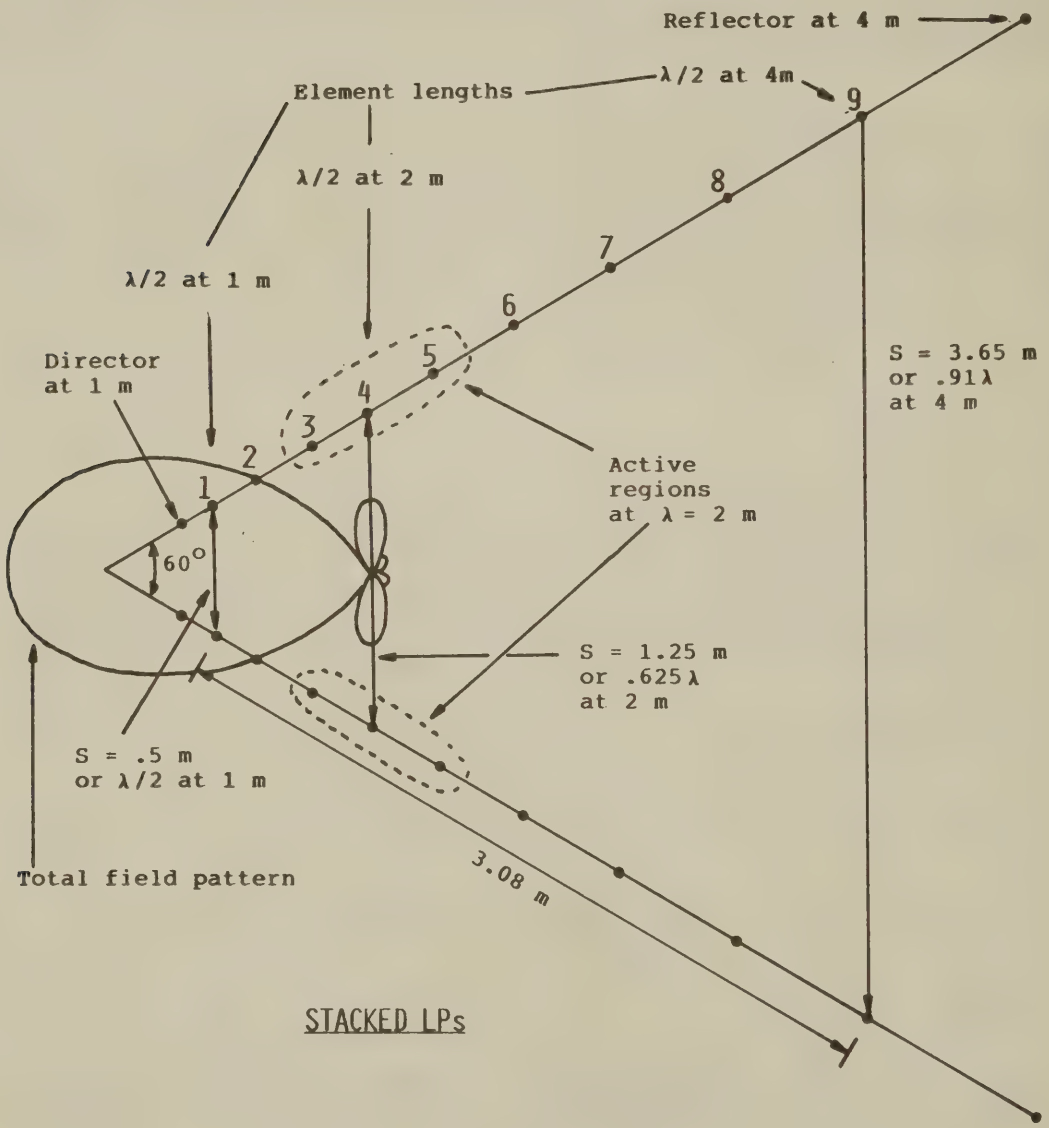
$$+ 0.387 + 0.464 + 0.557 + 0.669 = 3.08 \text{ m}$$

continued

15-3 continued

The stacked LPs are shown in side view in the sketch below. Elements 1 through 9 are included in the calculation. Element 1 is λ/2 resonant at 1 m wavelength and element 9 is λ/2 resonant at 4 m wavelength. A director element is added ahead of element 1 and a reflector element is added after element 9 making a total of 11 elements.

In the 60° angle stacking arrangement the stacking distance is .5 m or λ/2 at 1 m wavelength and 3.65 m or .91λ at 4 m wavelength. At the geometric mean wavelength (2 m) the stacking distance is 1.25 m or .625λ.



STACKED LPs

Let us calculate the vertical plane pattern at 2 m wavelength where elements 3, 4 and 5 of the upper and lower LPs are active. As an approximation, let us consider that the 3 active elements are a uniform ordinary end-fire array with spacing equal to the average of the spacing between elements 3 and 4 and between 4 and 5.

For element 4 we take $\ell_4 = \lambda/2$. Then from (15-5-6)

$$S_{45} = \frac{k - 1}{4 \tan \alpha} = \frac{1.2 - 1}{4 \tan 15^\circ} = .187\lambda$$

and $\quad S_{34} = .187/1.2 = .155\lambda$

$$S_{av} = \frac{.155 + .187}{2} = .171\lambda$$

The end-fire array field pattern is given by

$$E = \frac{1}{3} \frac{\sin n\psi/2}{\sin \psi/2}$$

where $\quad \psi/2 = \dfrac{2\pi \times .171}{2} (\cos \phi - 1) = 30.8^\circ (\cos \phi - 1)$

$n = 3$

Each LP (end-fire array) has a broad cardiod-shaped pattern like the ones shown in Fig. 15-14a with one pattern directed up 30° and the other down 30°.

The total field pattern in the resultant of these patterns and a broadside array of 2 in-phase isotropic sources stacked vertically and spaced .625λ with pattern given by

$$E = \cos [(2\pi \times .625/2) \sin \phi] = \cos (112.5^\circ \sin \phi)$$

This pattern is shown in Fig. 11-11. Numerical addition of the LP patterns and multiplication of the resultant by the broadside pattern yields the total field pattern for λ = 2 m shown in the sketch. At λ = 1 m the up-and-down minor lobes disappear but the main beam is about the same. At λ = 4 m the main beam is narrower but the up-and-down minor lobes are larger.

continued

15-3 continued

(b) Each LP has a gain $\cong$ 7 dBi. From the equation of Prob. 4-10, the directivity of 2 in-phase isotropic sources with $.625\lambda$ spacing is 2.44 or 3.9 dBi. So for λ = 2 m, the gain

may be as much as 7 + 3.9 $\cong$ $\boxed{\text{10.9 dBi}}$

At both λ = 1 m and λ = 4 m the gain may be less than this.

The HPBW in the vertical plane is about 47° and in the horizontal plane about 76°. From the approximate directivity relation, we have, neglecting minor lobes,

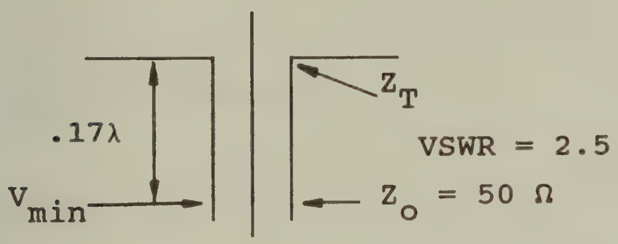
$$D = \frac{41\ 000}{47 \times 76} = 11.5 \text{ or } \boxed{10.6 \text{ dBi}}$$

Actual directivity is probably $\sim$ 10 dBi.

CHAPTER 16

ANTENNAS FOR SPECIAL APPLICATIONS: FEEDING CONSIDERATIONS

***16-2 Stub impedance.**



From (16-11-1),

$$Z_m = Z_o \frac{Z_T + jZ_o \tan \beta x}{Z_o + jZ_T \tan \beta x} \qquad (1)$$

where Z_m = impedance on line at $V_{min} = R_m + j0$

Z_o = line impedance = $50 + j0\ \Omega$

Z_T = stub antenna terminal impedance = $R_T + jX_T$

Rearranging (1) in terms of real and imaginary parts:

$$R_m - R_T = (X_T R_m / R_o) \tan \beta x \quad \text{by equating reals,} \qquad (2)$$

and

$$\frac{R_T R_m}{R_o} \tan \beta x = X_T + R_o \tan \beta x \quad \text{by equating imaginaries} \qquad (3)$$

$R_m = 50/2.5 = 20$, $R_o = 50$, $\tan \beta x = \tan (360^\circ \times .17) = 1.82$

From (2), $20 - R_T = X_T\ 20/50 \times 1.82 = .728\ X_T$

From (3), $R_T(20/50) \times 1.82 = X_T + 50 \times 1.82$; $.728\ R_T = X_T + 91$

From which, $Z_T = R_T + jX_T = \boxed{56 - j50\ \Omega}$

16-3 Square loop.

Squarish pattern with rounded edges. Maximum-to-minimum field ratio = 1.14.

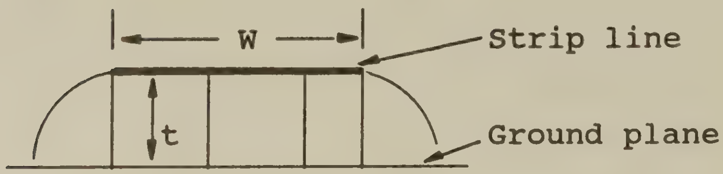
16-5 **Microstrip line.**

From (16-12-4) (see Fig. 16-32),

$$Z_c = \frac{Z_o}{\sqrt{\epsilon_r}\,[(W/t) + 2]} \qquad \text{or} \qquad \frac{W}{t} = \frac{Z_o}{Z_c\sqrt{\epsilon_r}} - 2$$

and

$$\frac{W}{t} = \frac{377}{50\sqrt{2.7}} - 2 = \boxed{2.6}$$



2.6 field cells under strip plus 2 fringing cells = 4.6 cells giving

$$Z_c = \frac{377}{\sqrt{2.7}\ \times 4.6} = 50\ \Omega$$

---

*16-6 **Surface-wave powers. Poynting vector.**

(a) $$S_{||\text{ to sheet}} = \frac{E_y^2}{Z_o} = \frac{100^2}{377} = \boxed{26.5\ \text{Wm}^{-2}}$$

(b) $$S_{\text{into sheet}} = H^2 R_e Z_c = \frac{E^2}{Z_o^2} R_e Z_c = (100/377)^2\ \frac{(3.7 \times 10^{-3})}{\sqrt{2}}$$

$$= \boxed{182\ \mu\text{W m}^{-2}}$$

16-7 **Surface-wave powers. Poynting vector.**

(a) $$S_{||\text{ to sheet}} = \frac{E_y^2}{Z_o} = \frac{150^2}{377} = \boxed{59.7\ \text{Wm}^{-2}}$$

16-7 continued

(b) $$S_{\text{into sheet}} = H^2 R_e Z_c = \left[\frac{E}{Z_o}\right]^2 R_e Z_c = (150/377)^2 \frac{(2 \times 10^{-2})}{\sqrt{2}}$$

$$= \boxed{2.24 \text{ mW m}^{-2}}$$

---

*16-8 Surface-wave power.

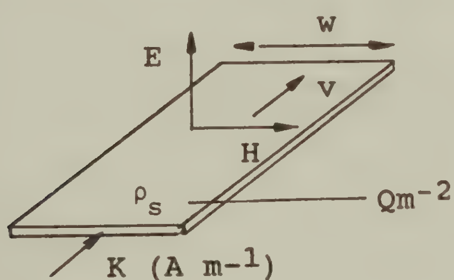
$$S_{\text{into sheet}} = H^2 R_e Z_c = \left[\frac{E}{Z_o}\right]^2 R_e Z_c$$

$$R_e Z_c = \sqrt{(\mu_o \omega/2\sigma)} = \left[\frac{4\pi \times 10^{-7}\ 2\pi \times 3 \times 10^9}{2 \times 10^7}\right]^{1/2} = .034\ \Omega$$

Therefore,

$$S_{\text{into sheet}} = (.075/377)^2 \times 0.034 = \boxed{1.35 \text{ nW m}^{-2}}$$

16-9 Surface wave current sheet.



$$K = \rho_s v = \frac{Q}{m^2}\frac{m}{s} = \frac{Q}{s}\frac{1}{m}$$

$$= A\ m^{-1} = I\ m^{-1}$$

By Amperes' law, integral of H around strip of width w equals current enclosed or

$$\oint H \cdot ds = I = wK$$

$wH = wK$ and $\boxed{H = K}$ Note that $H \perp K$

***16-11** **Coated-surface wave cutoff.**

$$\alpha = \frac{2\pi}{\lambda_o} \sqrt{(3-1)} = \boxed{8.89/\lambda_o \text{ Np m}^{-1}} \qquad f_c = \boxed{0}$$

***16-14** **Overland TV for HP, VP and CP.**

(a) and (b) answers in App. E.

(c) The effect of reflection from other buildings or structures (or from aircraft) can be minimized by the use of CP transmit and receive antennas of the same hand, particularly when these structures are many wavelengths in size and reflection is specular. Trouble-some reflections can be reduced by placing non-reflecting absorbers on the structure.

16-15 **Horizontal dipole above imperfect ground.**

$$\mu = \mu_o, \quad \epsilon_r' = 12, \ \sigma = 2 \times 10^{-3} \ \mho \text{ m}^{-1}, \quad h = \lambda/4$$

$$\rho_\perp = \frac{\sin\alpha - \left[\epsilon_r - \cos^2\alpha\right]^{1/2}}{\sin\alpha + \left[\epsilon_r - \cos^2\alpha\right]^{1/2}} \tag{1}$$

$$E_\perp = 1 + \rho_\perp \ \underline{/\cos(2\beta h \sin\alpha) + j\sin(2\beta h \sin\alpha)} \tag{2}$$

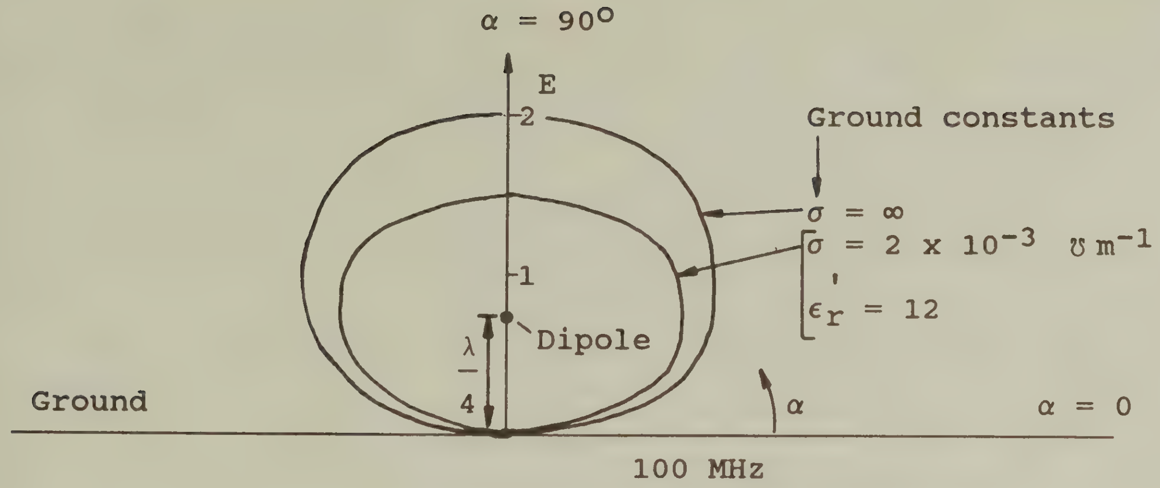
(b)

$$\epsilon_r'' = \frac{\sigma}{\omega\,\epsilon_o} = \frac{2 \times 10^{-3}}{2\pi\ 10^8\ 8.85 \times 10^{-12}} = 0.36 \text{ at } 100 \text{ MHz}$$

$$\epsilon_r = \epsilon_r' - j\,\epsilon_r'' = 12 - j0.4 \cong 12$$

Introducing ϵ_r into (1), (1) into (2) and evaluating (2) as a function of α results in the pattern shown. The pattern for perfectly conducting ground ($\sigma = \infty$) is also shown for comparison (same as pattern of 2 isotropic sources in phase opposition and spaced $\lambda/2$). For perfectly conducting ground the field doubles ($E = 2$) at the zenith ($\alpha = 90^o$) but with the actual ground of the problem, it is reduced to about 1.55 (down 2.2 dB) because of partial absorption of the wave reflected from the ground.

16-15 continued



(a) At 100 kHz, $\epsilon_r'' = 360$ and $\rho_\perp \cong -1$

so the pattern is approximately the same as for $\sigma = \infty$ in the sketch.

---

*16-17 DF and monopulse.

(a) $\alpha = 40^\circ$

$$\Delta\theta = 5^\circ, \quad \frac{P_2}{P_1} = \frac{\cos^4 (20^\circ - 5^\circ)}{\cos^4 (20^\circ + 5^\circ)} = 1.290 \text{ or } \boxed{1.1 \text{ dB}}$$

$$\Delta\theta = 10^\circ, \quad \frac{P_2}{P_1} = \frac{\cos^4 (20^\circ - 10^\circ)}{\cos^4 (20^\circ + 10^\circ)} = 1.672 \text{ or } \boxed{2.2 \text{ dB}}$$

(b) $\alpha = 50^\circ$

$$\Delta\theta = 5^\circ, \quad \frac{P_2}{P_1} = \frac{\cos^4 (25^\circ - 5^\circ)}{\cos^4 (25^\circ + 5^\circ)} = 1.386 \text{ or } \boxed{1.4 \text{ dB}}$$

$$\Delta\theta = 10^\circ, \quad \frac{P_2}{P_1} = \frac{\cos^4 (25^\circ - 10^\circ)}{\cos^4 (25^\circ + 10^\circ)} = 1.933 \text{ or } \boxed{2.9 \text{ dB}}$$

continued

*16-17 continued

(c) $\alpha = 5^\circ$, $\dfrac{P_0}{P_1} = \dfrac{\cos^4 0^\circ}{\cos^4 5^\circ} = 1.015$ or $\boxed{0.06 \text{ dB}}$

$\alpha = 10^\circ$, $\dfrac{P_0}{P_1} = \dfrac{\cos^4 0^\circ}{\cos^4 10^\circ} = 1.063$ or $\boxed{0.26 \text{ dB}}$

(d) Over 1 dB more at 5° and about 2 dB more at 10°.

---

*16-21 Signalling to submerged submarines.

From Table A-6, take $\epsilon_r' = 80$ and $\sigma = 4$ for sea water. At the highest frequency (1000 kHz), $\sigma >> \omega\epsilon$, so that $\alpha = \sqrt{(\omega\mu\sigma/2)}$ can be used at all four frequencies.

At 1 kHz,

$$\alpha = [(2\pi\ 10^3\ 4\pi \times 10^{-7} \times 4)/2)]^{1/2} = 0.13 \text{ Np m}^{-1}$$

Since

$$\frac{E}{E_o} = 10^{-6} = e^{-\alpha y}, \quad y = \frac{6}{\alpha} \log e = \frac{13.8}{\alpha}$$

and at 1 kHz, depth $y = \boxed{106 \text{ m}}$

at 10 kHz, $y = \boxed{35 \text{ m}}$

at 100 kHz, $y = \boxed{11 \text{ m}}$

at 1000 kHz, $y = \boxed{3.5 \text{ m}}$

From the standpoint of frequency, 1 kHz gives greatest depth. However, from (16-2-3) the radiation resistance of a monopole antenna as a function of its height (h_p) is

$$R_r = 400\ (h_p/\lambda)^2 \quad (\Omega)$$

***16-21** continued

For h_p = 300 m at 1 kHz

$$R_r = 400\ (300/3 \times 10^5)^2 = 4 \times 10^{-4}\ \Omega \quad (\text{or } 400\ \mu\Omega)$$

with such a small radiation resistance, radiation efficiency will be poor. At 10 kHz the radiation resistance is a hundred times greater. A practical choice involves a compromize of sea water loss, land (transmitting) antenna effective height, and submarine antenna efficiency as a function of the frequency.

CHAPTER 17

ANTENNA TEMPERATURE, REMOTE SENSING, RADAR & SCATTERING

*17-1 Satellite TV downlink.

(a) Satellite ERP = 35 dB (over 1 W isotropic)

$ERP = P_t D_t = P_t \; 4\pi \; A_{et}/\lambda^2$

$P_t A_{et} = \lambda^2 \; ERP/4\pi$

From (17-3-9)

$$\frac{S}{N} = \frac{P_t A_{et} A_{er}}{kT_{sys} r^2 \lambda^2 \Delta f_r} = \frac{\lambda^2 \; ERP \; A_{er}}{4\pi \; kT_{sys} r^2 \lambda^2 \Delta f_r}$$

ERP = 35 dB or 3162

$$A_{er} = \frac{1}{2} \pi \; r^2 = \frac{1}{2} \pi \; 1.5^2 = 3.53 \text{ m}^2$$

$T_{sys} = 25 + 75 = 100$ K

$$\frac{S}{N} = \frac{3162 \times 3.53}{4\pi \times 1.38 \times 10^{-23} \times 100 \times 3.6^2 \times 10^{14} \times 3 \times 10^7}$$

$$= 16.6 \quad \text{or} \quad \boxed{12.2 \text{ dB}}$$

(b) If only 10 dB S/N ratio is acceptable, the dish aperture could be 2.2 dB (= 12.2 - 10 dB) less or 60% of the $\pi \; 1.5^2$ = 7.07 m^2 area specified in part (a).

Therefore acceptable area = 7.07 x .6 = 4.24 m^2 = πr^2

for a diameter = $2r = 2\;(4.24/\pi)^{1/2} = \boxed{2.3 \text{ m}}$

*17-2 Antenna temperature.

$0.9\Omega_A$ is at 5K

$0.08\Omega_A$ is at 50K

Therefore, $0.02\Omega_A$ is at 300K

From (17-2-9),

$$T_A = \frac{1}{\Omega_A}[\ 5 \times 0.9\Omega_A + 50 \times 0.08\Omega_A + 300 \times 0.02\Omega_A]$$

$$= 4.5 + 4 + 6 = \boxed{14.5\ \text{K}}$$

17-3 Earth-station antenna temperature.

From (17-2-9)

$$T_A = \frac{1}{\Omega_A}[\ 6 \times 0.8\Omega_A + 6 \times \frac{2}{3} \times 0.2\Omega_A + 300 \times \frac{1}{3}\ 0.2\Omega_A]$$

$$= 4.8 + 0.8 + 20 = \boxed{25.6\ \text{K}}$$

---

*17-4 System temperature.

(a) Noise output readings 234, 235, etc. with respect to average value (231), are squared, averaged and then square rooted for root-mean-square (rms) noise value 4.71.

The rms noise at receiver is then

$$4.71/170 \times 2.9 = \boxed{0.08\ \text{K}}$$

(b) Transmission line attenuation = 0.5 dB for efficiency of 0.89.

Therefore, $\Delta T_{min} = 0.08/0.89 = \boxed{0.09\ \text{K}}$

continued

*17-4 continued

(c) From (17-3-3)

$$T_{sys} = \frac{\Delta T_{min}(\Delta f t)^{1/2}}{k'} = \frac{0.09\ (7 \times 10^6 \times 14)^{1/2}}{2}$$

$$= \boxed{445\ \mathrm{K}}$$

(d) From (17-2-8)

$$\Delta S_{min} = \frac{2k\ \Delta T_{min}}{A_e} = \frac{2 \times 1.38 \times 10^{-23} \times 0.09}{500}$$

$$= 497\ \mathrm{mJy}, \quad \text{rounded off,} \quad = \boxed{500\ \mathrm{mJy}}$$

17-5 **System temperature.**

From (17-3-1) and (17-3-2)

$$T_{sys} = 15 + 300 \left[\frac{1}{.95} - 1\right] + \frac{1}{.95}\left[75 + \frac{100}{40} + \frac{200}{40}\right]$$

$$= 15 + 15.8 + 78.9 + 2.6 + 5.3 = \boxed{117.6\ \mathrm{K}}$$

---

*17-6 **Minimum detectable temperature.**

(a) For 1 dB loss, $\epsilon_2 = 1/1.26 = .79$

From (17-3-1)

$$T_{sys} = 50 + 270 \left[\frac{1}{.79} - 1\right] + \frac{50}{.79}$$

$$= 50 + 71.8 + 63.3 = 185\ \mathrm{K}$$

*17-6 continued

From (17-3-3), $\Delta T_{min} = k' T_{sys}/(\Delta f\ t\ n)^{1/2}$

where n = number of records averaged

$$= \frac{\pi}{\sqrt{2}} [185/(5 \times 10^6 \times 5 \times 2)^{1/2}]$$

$$= 0.058 \text{ K}, \quad \text{rounded off} = \boxed{0.06 \text{ K}}$$

(b) From (17-2-8)

$$\Delta S_{min} = 2\, k \Delta T_{min} / A_e = \frac{2 \times 1.38 \times 10^{-23} \times 0.058}{500} = \boxed{320 \text{ mJy}}$$

17-7 Minimum detectable temperature.

(a) $$\Delta T_{min} = \frac{2.2 \times 150}{(10^8 \times 5)^{1/2}} = \boxed{0.015 \text{ K}}$$

(b) $$\Delta S_{min} = \frac{2 \times 1.38 \times 10^{-23} \times 0.015}{800} = \boxed{52 \text{ mJy}}$$

(c) $$\Delta T_{min} = 0.015/\sqrt{4} = \boxed{0.008 \text{ K}}$$

$$\Delta S_{min} = 52/\sqrt{4} = \boxed{26 \text{ mJy}}$$

---

*17-8 Antenna temperature with absorbing cloud.

From (17-4-1) and (17-2-7)

$$\Delta T_A = \frac{\Omega_c}{\Omega_A} T_c (1 - e^{-\tau_c}) + \frac{\Omega_s}{\Omega_A} T_s\, e^{-\tau_c}$$

continued

*17-8 continued

$$\Omega_c = 5/57.3^2 = 0.00152 \text{ sr}, \quad \Omega_s = 1/57.3^2 = 0.00030 \text{ sr}$$

$$\Omega_A = \lambda^2/A_{em} = 0.5^2/50 = 0.005 \text{ sr}$$

Therefore,

$$\Delta T_A = \frac{.00152}{.005} 100 (1-e^{-1}) + \frac{.0003}{.005} 200 e^{-1} = \boxed{23.6 \text{ K}}$$

17-10 Forest absorption.

From (17-4-2)

$$e^{-\tau_f} = \frac{\Delta T_A - T_f}{T_f - T_e} = \frac{294 - 288}{300 - 288} = 0.5 \quad \text{and} \quad \tau_f = \boxed{0.693}$$

---

*17-11 Solar interference to earth station.

(a) Earth station $S_{min} = \dfrac{2kT_{sys}}{A_e}$

$$S_{sun} = \frac{2k\Delta T_A}{A_e} = \frac{2k\Delta T_A \Omega_A}{A_e \Omega_A} = \frac{2k\ T_s \Omega_s}{\lambda^2}$$

Assuming 50% aperture efficiency as in Prob. 17-1,

$$\Omega_A = \frac{\lambda^2}{A_e} = \frac{0.075^2}{\frac{1}{2}\pi\ 1.5^2} = 1.59 \times 10^{-3} \text{ sr} = 5.22 \text{ sq. deg.}$$

Therefore,

$$\frac{S}{N} = \frac{S_{sun}}{S_{min}} = \frac{T_s \Omega_s}{T_{sys} \Omega_A} = \frac{5 \times 10^4 \times \pi\ (.25^\circ)^2}{100 \times 5.22}$$

sq. deg.

$$= 18.8 \quad \text{or} \quad \boxed{12.7 \text{ dB}}$$

*<u>17-11</u> continued

(b) From Prob. 17-1

12.7-12.2 = 0.5 dB more than satellite carrier, resulting in degradation of TV picture quality. The phenomenon may be described as <u>noise jamming</u> by the sun.

(c) At half-power, solar noise will be reduced to only 3 - 0.5 = 2.5 dB below carrier. Assuming low side-lobes, the solar interference should not last more than the time it takes the sun to drift between first nulls. Using this criterion we have

$$\text{Time} = \underbrace{4 \text{ (min/deg)}}_{\text{Solar drift rate}} \times \text{BWFN (deg)}$$

From Table A-10, $\text{HPBW} = 66^\circ/D_\lambda = 66^\circ/(3/.075) = 1.65^\circ$

and $\text{Time} \cong 4 \times 2 \times 1.65 = \boxed{13.2 \text{ min}}$

Allowing for the angular extent of the sun (approx. $\frac{1}{2}^\circ$), increases the time by about 2 minutes.

<u>17-12</u> Radar detection.

From the radar equation (17-5-5)

$$P_t = P_r \frac{(4\pi)^3 r^4}{G^2 \lambda^2 \sigma} = \frac{10^{-12} (4\pi)^3 (10^3)^4}{(4\pi/\lambda^2)^2 \lambda^2 \times 5} = \frac{4\pi \times 0.1^2}{5} = \boxed{25 \text{ mW}}$$

---

*<u>17-16</u> Jupiters signals.

$r = 40$ light min $\times 60$ s min$^{-1} \times 3 \times 10^8$ m s$^{-1} = 7.20 \times 10^{11}$ m

$$\frac{P_r}{\Delta f} = \frac{P_t}{\Delta f} \times \frac{1}{4\pi r^2} \quad \text{or}$$

$$\frac{P_t}{\Delta f} = \frac{P_r}{\Delta f} \times 4\pi r^2 = 10^{-20} \times 4\pi \times 7.2^2 \times 10^{22} = \boxed{65.1 \text{ kW Hz}^{-1}}$$

*<u>17-18</u> **Critical frequency. MUF.**

MUF = Maximum Usable Frequency for communication via ionospheric reflection.

(a) $f_o = 9\sqrt{N} = 9\ (6 \times 10^{11})^{1/2} = 6.97$ MHz

$$\phi = \tan^{-1}\ [(d/2)/h] = \tan^{-1}\left[\frac{6.5 \times 10^5}{3.25 \times 10^5}\right] = 63.43^{\circ}$$

$\cos\phi = 0.447$

$\text{MUF} = f_o/\cos\phi = 6.97/0.447 = \boxed{15.6 \text{ MHz}}$

(b) $f_o = 9\ (10^{12})^{1/2} = 9$ MHz

$$\phi = \tan^{-1}\left[\frac{7.5 \times 10^5}{2.75 \times 10^5}\right] = 69.86^{\circ}, \qquad \cos\phi = 0.344$$

$\text{MUF} = 9/0.344 = \boxed{26.1 \text{ MHz}}$

(c) $f_o = 9(8 \times 10^{11})^{1/2} = 8.05$ MHz

$$\phi = \tan^{-1}\left[\frac{5 \times 10^5}{10^5}\right] = 78.69^{\circ}, \qquad \cos\phi = 0.196$$

$\text{MUF} = 8.05 \times 10^6/0.196 = \boxed{41.0 \text{ MHz}}$

17-19 mUF for Clarke-orbit satellites.

mUF = minimum usable frequency for transmission through ionosphere

(a) $\text{mUF} = 9(N)^{1/2} = 9(10^{12})^{1/2} = \boxed{9\text{ MHz}}$

Although MUF = mUF, whether it is one or the other, depends on the point of view. It is MUF for reflection and mUF for transmission. Below critical frequency, wave is reflected; above critical frequency, wave is transmitted.

(b) $\text{mUF} = 9\text{ MHz}/\cos 30^\circ = \boxed{10.4\text{ MHz}}$

(c) $\text{mUF} = 9\text{ MHz}/\cos 75^\circ = \boxed{34.8\text{ MHz}}$

A typical Clarke orbit satellite frequency is 4 GHz which is 115 ($= 4 \times 10^9/34.8 \times 10^6$) times higher in frequency than the mUF so that at 4 GHz, transmission through the ionosphere can occur at much lower elevation angles than 15°.

17-23 Effect of resonance on radar cross section of short dipoles.

(a) $$\boxed{0.714\lambda^2} = \underbrace{4 \times \overbrace{0.119}^{A_{em}\ \text{(matched dipole, } Z_L = Z_A^*)}}_{A_s = \sigma_t\ \text{(resonant dipole, } Z_L = -jX_A)} \times \underbrace{1.5}_{\text{Directivity}}\ \lambda^2$$

(b) $\boxed{2 \times 10^{-5}\ \lambda^2}$

(c) $\boxed{0.83\ \lambda^2}$

continued

Summary and comparison:

Case	σ	Condition
(a)	$0.714\lambda^2$	Resonant short dipole, length 0.1λ ($Z_L = -jX_A$)
(b)	$2 \times 10^{-5}\lambda^2$	Non-resonant 0.1λ dipole ($Z_L = 0$)
(c)	$0.83\lambda^2$	Resonant 0.47λ dipole ($Z_L = 0$)

In (a) resonance is obtained by making $Z_L = -jX_A$. In (c) resonance is obtained ($X_A = 0$) by increasing the length to 0.47λ (with $Z_L = 0$, terminals short-circuited).

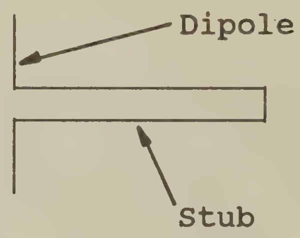
The cross section in (c) is larger than in (a) because the dipole is physically longer.

In (b) the dipole is non-resonant because $Z_L = 0$ (terminals short-circuited) and the length (0.1λ) is much less than the resonant length (0.47λ), resulting in a very small radar cross section.

It appears that if a short dipole (length $\leq 0.1\lambda$) is resonated by making $Z_L = -jX_A$, its radar cross section ($0.714\lambda^2$) approaches the cross section of a resonant $\lambda/2$ dipole (length = 0.47λ), regardless of how short it is, provided it is lossless.

A short dipole may be resonated ($Z_L = -jX_A$) by connecting a stub of appropriate length across its terminals,

thus,



or by connecting a lumped inductance,

thus,

CHAPTER 18

ANTENNA MEASUREMENTS

*<u>18-1</u> **Absorbing materials. 1/e depths.**

$$\delta(1/e) = 1/\sqrt{f\pi\mu\sigma} = \frac{1}{\sqrt{f}(\pi \times 4\pi \times 10^{-7} \times 2 \times 10^2)^{1/2}} = \frac{35.6}{\sqrt{f}}$$

provided $\sigma \gg \omega\epsilon$.

$\sigma = 10^2 \ \mho\, m^{-1}$

$\epsilon = \epsilon_r \epsilon_o = 3 \times 8.85 \times 10^{-12} \ \ F\, m^{-2}$

At all 3 frequencies of problem, the condition $\sigma \gg \omega\epsilon$ is satisfied.

(a) $\delta(1/e) = 35.6/\sqrt{60} = \boxed{4.60 \text{ m}}$ $\qquad \delta(1\%) = \boxed{21.2 \text{ m}}$

where $\delta(1\%) = 4.61 \times \delta(1/e)$

(b) $\delta(1/e) = 35.6/(2 \times 10^6)^{1/2} = \boxed{25.2 \text{ mm}}$ $\qquad \delta(1\%) = \boxed{116 \text{ mm}}$

(c) $\delta(1/e) = 35.6/(3 \times 10^9)^{1/2} = \boxed{650\ \mu\text{m}}$ $\qquad \delta(1\%) = \boxed{3.00 \text{ mm}}$

<u>18-2</u> **Lossy medium. Wave absorption.**

$$\gamma = j\frac{2\pi}{\lambda_o}\sqrt{\mu_r\epsilon_r} = j\frac{2\pi}{\lambda_o}[15(1-j3)\ 50\ (1-j1)]^{1/2} = 243 \ \underline{/32^\circ}$$

$$\gamma = 206 + j128 = \alpha + j\beta$$

(a) $$\frac{Z}{Z_o} = (\mu_r/\epsilon_r)^{1/2} = [15(1-j3)/50(1-j1)]^{1/2}$$

$$= 0.80 - j0.19 = 0.82 \ \underline{/-13.3^\circ}$$

continued

18-2 continued

(b) $\lambda/\lambda_o = 2\pi/\beta\lambda_o = 2\pi/(128 \times 1.5) = \boxed{0.033}$ $\lambda_o = 1.5$ m

(c) $v/v_o = \lambda/\lambda_o = \boxed{0.033}$

(d) $\delta = 1/\alpha = 1/206 = \boxed{4.9\ \text{mm}}$

(e) $x/\delta = 5/4.9 = 1.03$

Attenuation $= e^{-1.03} = 0.357$ or $\boxed{8.95\ \text{dB}}$

(f) $$\rho = \frac{(Z/Z_o)\ -1}{(Z/Z_o)\ +1} = \frac{0.80 - j0.19 - 1}{0.80 - j0.19 + 1} = \boxed{0.152\ \angle -130^o}$$

Thus, a wave incident on this medium is largely absorbed(reflected wave down ≅ 16 dB).

---

*18-3 Lossy medium. Complex constants.

$$\alpha = \frac{2\pi}{\lambda_o} \text{Re}\ j\{[\epsilon_r' - j(\epsilon_r'' + \sigma/\omega\epsilon_o)]^2\}^{1/2}$$

$$= \frac{2\pi}{\lambda_o}(\epsilon_r'' + \sigma/2\pi f\epsilon_o) \qquad \lambda_o = 0.01 \text{ m}$$

$$= \frac{2\pi}{.01}\left(2 + \frac{3.34}{1.668}\right) = 800\pi$$

$$\delta\ (1/e) = 1/\alpha = 1/800\pi = \boxed{398\ \mu\text{m}}$$

18-4 Attenuation by lossy slab. Internal attenuation.

$\lambda_o = c/f = 0.5$m $\qquad \gamma = j\dfrac{2\pi}{\lambda_o}\sqrt{\mu_r\epsilon_r} = j\dfrac{2\pi}{\lambda_o}(2 - j2)$

$\alpha = 4\pi/\lambda_o = 8\pi \qquad e^{-\alpha x} = e^{-8\pi(.2)} = e^{-5.03} = .00654$

18-4 **continued**

dB attenuation = 20 log (1/.00654) = $\boxed{43.7 \text{ dB}}$

or $\underset{\text{nepers}}{5.03} \times \underset{\text{dB/neper}}{8.686} = 43.7$ dB

***18-5** **Attenuation of lossy sheet.**

$\lambda_o = 3 \times 10^8 / 3 \times 10^8 = 1$ m $\qquad \gamma = j \dfrac{2\pi}{\lambda_o} \sqrt{\mu_r \epsilon_r} = j \dfrac{2\pi}{\lambda_o} (5 - j5)$

$\alpha = 10\pi/\lambda_o \qquad e^{-\alpha x} = e^{-10\pi(.006)} = e^{-.1885} = .828$

dB attenuation = 20 log (1/.828) = $\boxed{1.64 \text{ dB}}$

18-6 **Attenuation of conducting sheet.**

Involves loss on entering, loss inside and loss on leaving sheet.

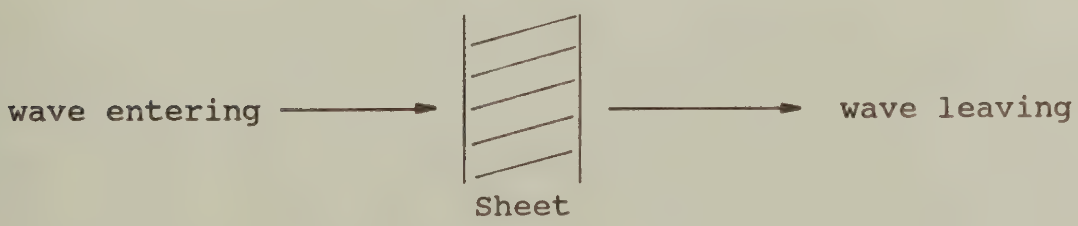
$\delta = 1/(\pi f \mu \sigma)^{1/2}$ provided $\sigma >> \omega\epsilon$

$\sigma = 10^3$ ℧ $m^{-1} \qquad \omega\epsilon = 2\pi \times 8 \times 10^8 \times 8.85 \times 10^{-12} = 0.045$

so condition is satisfied that $\sigma >> \omega\epsilon$.

$\delta = 1/(\pi \times 8 \times 10^8 \times 4\pi \times 10^{-7} \times 10^3)^{1/2} = 1/1777$

$e^{-1777(.002)} = e^{-3.554} = .0286$ = attenuation within sheet



continued

18-6 continued

Amount of wave entering sheet:

$$Z_s = (\mu\omega/\sigma)^{1/2}\underline{/45^\circ} = (4\pi \times 10^{-7} \times 2\pi \times 8 \times 10^8/10^3)^{1/2}$$

$$= 2.51 \underline{/45^\circ}\ \Omega$$

Transmission coefficient on entering $= 2\,Z_s/(Z_s + Z_o) = \tau_v$

$$|\tau_v| \cong 2 \times 2.51/(2.51 + 377) = 0.013$$

Transmission coefficient on leaving $= 2\,Z_o/(Z_s + Z_o) = \tau_v$

$$|\tau_v| \cong 2 \times 377/(377 + 2.51) \cong 2$$

Therefore on leaving, E doubles.

$$\text{Total loss} = \underset{\text{Entering}}{0.013} \times \underset{\text{Inside}}{0.0286} \times \underset{\text{Leaving}}{2} = 7.55 \times 10^{-4}$$

dB attenuation from left to right of sheet $= 20 \log (10^4/7.55)$

$$= \boxed{62.4 \text{ dB}}$$

---

*18-7 Attenuation by lossy medium.

Given $\sigma = 1.112 \times 10^{-2}$ ℧ m^{-1} $\qquad \mu_r = 5 - j4$

$\epsilon_r' = 5 \qquad \epsilon_r'' = 2 \qquad f = 100$ MHz

$$\epsilon_r = \epsilon_r' - j(\epsilon_r'' + \sigma/\omega\epsilon_o) = 5 - j4$$

(a) Therefore $Z = 377\,(\mu_r/\epsilon_r)^{1/2} = 377 \left[\dfrac{5 - j4}{5 - j4}\right]^{1/2} = \boxed{377\ \Omega}$

(b) $\gamma = j\,\dfrac{2\pi}{\lambda_o}\,[(5 - j4)^2]^{1/2} = j\,\dfrac{2\pi}{\lambda_o}\,(5 - j4)$

$\alpha = 8\pi/\lambda_o \qquad \lambda_o = c/f = 3 \times 10^8/10^8 = 3\text{m}$

*18-7 continued

$$e^{-\alpha x} = 0.1, \text{ corresponding to 20 dB attenuation}$$

$$-\alpha x = \ln 0.1 = -2.30$$

$$x = 2.30/\alpha = 2.30\ \lambda_o/8\pi = 2.30 \times 3/8\pi = \boxed{275 \text{ mm}}$$

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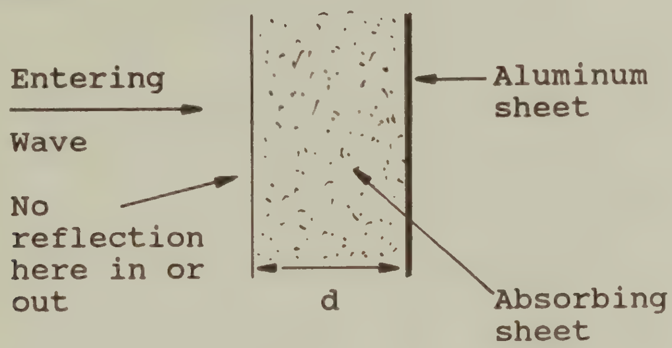
*18-8 Absorbing sheet.



$$Z = 377\,[(6 - j6)/(6 - j6)]^{1/2} = 377\ \Omega \qquad \lambda_o = \frac{3 \times 10^8}{5 \times 10^8} = 0.6 \text{ m}$$

Therefore, wave enters without reflection

$$\gamma = j\,\frac{2\pi}{\lambda_o}\,[(6 - j6)^2]^{1/2} = j\,\frac{2\pi}{\lambda_o}\,(6 - j6)$$

$$\alpha = 12\pi/\lambda_o$$

$$20 \log x = 30; \quad x = \text{antilog } 1.5 = 31.6$$

$$\text{Attenuation} = e^{-\alpha x} = e^{-\alpha 2d} = 1/31.6 \qquad -\alpha 2d = -3.45$$

$$d = \frac{1}{2}\,\frac{3.45\,\lambda_o}{12\pi} = \frac{3.45 \times .6}{2 \times 12\pi} = \boxed{27.5 \text{ mm}}$$

---

*18-9 Reflection from dielectric medium.

$$\text{Reflection coefficient} = \rho_V = \frac{Z_d - Z_o}{Z_d + Z_o}$$

continued

*18-9 continued

$$Z_d = 377\ (\mu_r/\epsilon_r)^{1/2} = 377\ [1/(2 - j2)]^{1/2} = 224\ \underline{/22.5^\circ}$$

$$Z_d/Z_o = \frac{224\ \underline{/22.5^\circ}}{377} = .594\ \underline{/22.5^\circ} = .55 + j.23$$

$$\rho_v = \frac{(Z_d/Z_o) - 1}{(Z_d/Z_o) + 1} = .32\ \underline{/145^\circ}$$

Reflected wave power $= .32^2 = .102$ or $\boxed{9.9 \text{ dB down}}$

18-10 CP wave relection and transmission.

(a) $Z_{med} = 377(\mu_r/\epsilon_r)^{1/2} = 377\ [(3 - j3)/(3 - j3)]^{1/2} = 377\ \Omega$

Therefore reflected wave PV = $\boxed{0}$

(b) $$\gamma = j\frac{2\pi}{\lambda_o}[(3 - j3)^2]^{1/2} = j\frac{2\pi}{\lambda_o}(3 - j3)$$

$\alpha = 6\pi/\lambda_o$ $\qquad$ $\lambda_o = c/f = \dfrac{3 \times 10^8}{2 \times 10^8} = 1.5$ m

$$\text{Attenuation} = e^{-\alpha x} = e^{-(6\pi/1.5)(.2)} = e^{-2.513} = 0.081$$

$$\text{Wave PV} = \frac{E^2}{Z_o} = \frac{2^2}{377} = 0.0106 \text{ Wm}^{-2}$$

PV at 200 mm depth $= .0106 \times .081^2 = 7 \times 10^{-5}$ W m^{-2}

$$= \boxed{70\ \mu\text{W m}^{-2}}$$

18-11 **Radar cross section.**

Assuming sphere radius = 5λ is enough larger than λ that formulas of Table 17-1 can be used. Thus, for a sphere

Radar cross section, $\sigma_s = \pi a^2$, $\quad a = 5\lambda$.

$$10\log x = 8 \text{ or } x = 6.3$$

Therefore, σ (object) $= \sigma_s \times 6.3 = \pi s \times 5^2 \times 6.3 = \boxed{495\ \lambda^2}$

18-12 **Aperture efficiency from Cygnus A.**

From App. A-7, the flux density of Cygnus A at 2.7 GHz is 785 Jy.

From (18-6-16),

$$A_e = \frac{2K\,\Delta T_A}{S} = \frac{2 \times 1.38 \times 10^{-23} \times 51}{785 \times 10^{-26}} = 179\ \text{m}^2$$

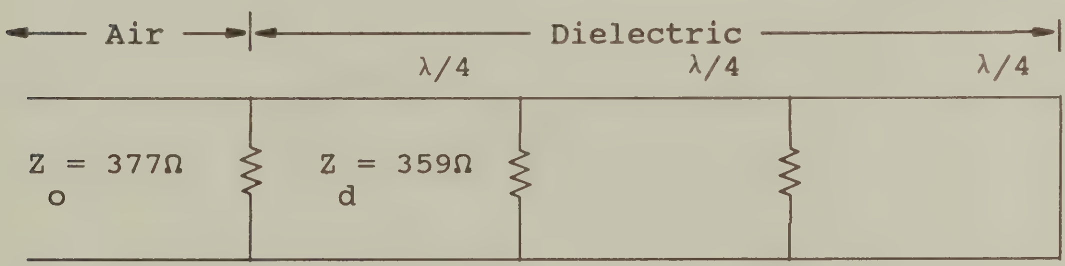
$$A_p = \pi\ 10^2 = 314\ \text{m}^2$$

$$\epsilon_{ap} = A_e/A_p = 179/314 = 0.57 \text{ or } \boxed{57\%}$$

18-13 **Jaumann sandwich absorber.**

Use either transmission line theory or a Smith chart as illustrated for $f/f_o = 1.5$.

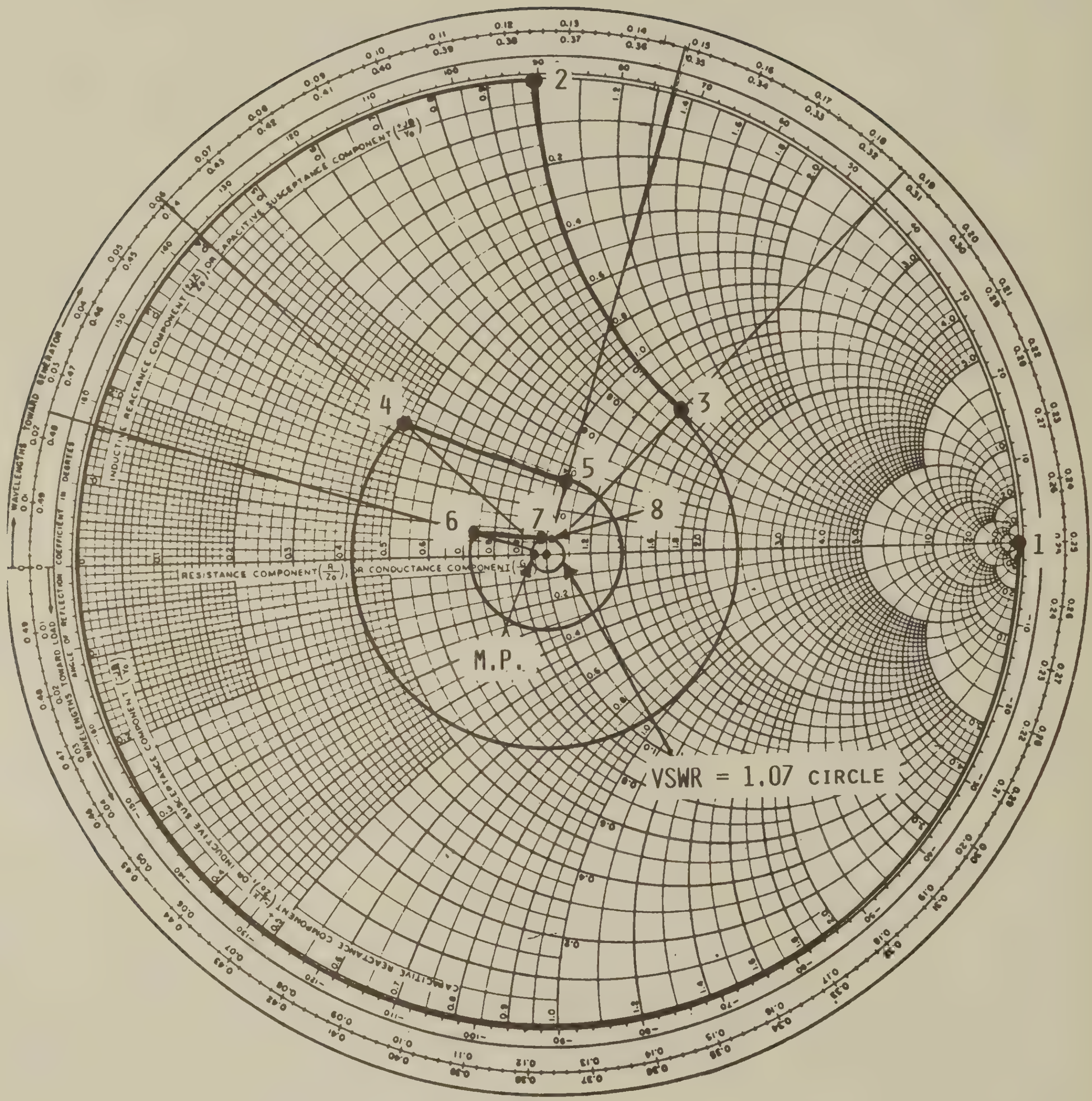
$$Z_d \text{ of dielectric medium} = \frac{377}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{1.1}} = 359\ \Omega \text{ per square}$$



R:	1563	625	250 Ω per square
$R_n = R/Z_d =$	4.35	1.74	0.70
$G_n = 1/R_n =$	.23	.57	1.4

continued

18-13 continued



18-13 continued

Enter Smith chart at $Y = \infty$ (short circuit) (point 1)

For $f/f_o = 1.5$ move $.25 \times 1.5 = .375\lambda$ (to point 2)

Add $G_n = 1.4$ (to point 3)

Move $.375\lambda$ (to point 4)

Add $G_n = .57$ (to point 5)

Move $.375\lambda$ (to point 6)

Add $G_n = .23$ (to point 7)

At point 7, $Y_n = .97 + j.08$

The center point of the chart (for the dielectric medium, $\epsilon_r = 1.1$) is at $Y_n = 1.0 + j0$. The air to the left of the sandwich has an intrinsic impedance

$$Z_o(\text{air}) = 377\ \Omega/\text{square}$$

and the dielectric sandwich to the right

$$Z_d(\text{dielectric}) = 359\ \Omega/\text{square}$$

Their admittance ratio is

$$\frac{Y_o(\text{air})}{Y_d(\text{dielectric})} = .95 + j0$$

so that the match point (M.P.) for the dielectric sandwich to match the air is at

$$Y_n = .95 + j0$$

Normalizing the chart to air

$$Y_n(\text{air}) = \frac{.97 + j.08}{.95} = 1.02 + j.084 \quad \text{(point 8)}$$

This is on the VSWR = 1.07 circle

continued

18-13 continued

Therefore reflection coefficient,

$$|\rho| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = 0.34 \quad \text{or 29 dB down}$$

In the same way obtain $|\rho|$ for other f/f_o values to determine bandwidth for which

$|\rho| \leq 0.10$ (20 dB or more down) Ans: $\boxed{\sim 3:1}$

The dielectric-to-air mismatch results in a reflection coefficient

$$|\rho| = \frac{377 - 359}{377 + 359} = 0.024 \text{ or 32 dB down}$$

Although the above analysis and that of Nortier, Van der Neut and Baker takes this into account, the effect is small.

Note that the calculation is for normal incidence.

Note also that the 6-resistance absorber of Fig. 18-12 has approximate exponential tapers in both resistance <u>and</u> spacing.

Nortier, Van der Neut and Baker determined the reflection coefficient using transmission line equations and a computer program.

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*Index reads as follows: Entry (Problem number) page

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